```
Off[General::"spell"] ; Off[General::"spell1"] ;
```

Put the original Skyrme interaction in to see that we get what we expect from the gradient terms:

```
P1 = t1 / 4 (1 + x1 / 2)
```

$$\frac{1}{4}\, t1 \left(1+\frac{x1}{2}\right)$$

```
P2 = t2 / 4 (1 + x2 / 2)
```

$$\frac{1}{4}\, t2 \left(1+\frac{x2}{2}\right)$$

```
Q1 = t1 / 4 (1 / 2 + x1)
```

$$\frac{1}{4}\, t1 \left(\frac{1}{2}+x1\right)$$

```
Q2 = t2 / 4 (1 / 2 + x2)
```

$$\frac{1}{4}\, t2 \left(\frac{1}{2}+x2\right)$$

```
P1f = P1
```

$$\frac{1}{4}\, t1 \left(1+\frac{x1}{2}\right)$$

```
P2f = P2
```

$$\frac{1}{4}\, t2 \left(1+\frac{x2}{2}\right)$$

```
dQ2dn = 0
```

0

```
Hgradient = Simplify[
  -1 / 4 (2 P1 + P1f - P2f) (nn[z] + np[z]) (nn''[z] + np''[z])
   + 1 / 2 (Q1 + Q2) (nn[z] nn''[z] + np[z] np''[z])
   - 1 / 4 (Q1 - Q2) (nn'[z]^2 + np'[z]^2)
   + dQ2dn / 2 (nn[z]' nn[z] + np[z] np'[z]) (nn'[z] + np'[z])]
```

$$\frac{1}{32}\,\big(-(t1+2\,t1\,x1-t2\,(1+2\,x2))\,(nn'[z]^2+np'[z]^2)-(3\,t1\,(2+x1)-t2\,(2+x2))\,(nn[z]+np[z])\,(nn''[z]+np''[z])+2\,(t1+t2+2\,t1\,x1+2\,t2\,x2)\,(nn[z]\,nn''[z]+np[z]\,np''[z])\big)$$

```
Hgradient2 = -1 / 16 (3 t1 (1 + x1 / 2) - t2 (1 + x2 / 2)) (nn[z] + np[z]) (nn''[z] + np''[z]) +
  1 / 16 (3 t1 (1 / 2 + x1) + t2 (1 / 2 + x2)) (nn[z] nn''[z] + np[z] np''[z])
```

$$-\frac{1}{16}\left(3\,t1\left(1+\frac{x1}{2}\right)-t2\left(1+\frac{x2}{2}\right)\right)(nn[z]+np[z])\,(nn''[z]+np''[z])+\frac{1}{16}\left(3\,t1\left(\frac{1}{2}+x1\right)+t2\left(\frac{1}{2}+x2\right)\right)(nn[z]\,nn''[z]+np[z]\,np''[z])$$

```
Simplify[
 (Hgradient - Hgradient2) /. nn′[z] → Sqrt[-nn[z] nn″[z]] /. np′[z] → Sqrt[-np[z] np″[z]]]
```

0

The original APR Lagrangian:

```
HAPR = (h^2 / 2 / m + (p3 + (1 - x) p5) ρ Exp[-p4 ρ]) τn + (ħ^2/2/m + (p3 + x p5) ρ Exp[-p4 ρ]) τp +
   (1 - (1 - 2 x)^2) (-ρ^2 (p1 + p2 ρ + p6 ρ^2 + (p10 + p11) Exp[-p9^2 ρ^2])) +
   (1 - 2 x)^2 (-ρ^2 (p12 / ρ + p7 + p8 ρ + p13 Exp[-p9^2 ρ^2]))
```

$$-(1-2x)^2 \rho^2 \left(e^{-p9^2 \rho^2}\, p13 + p7 + \frac{p12}{\rho} + p8\, \rho\right) - (1-(1-2x)^2)\, \rho^2 \left(p1 + e^{-p9^2 \rho^2} (p10 + p11) + p2\, \rho + p6\, \rho^2\right) + \left(\frac{h^2}{2\, m} + e^{-p4\, \rho} (p3 + p5\, (1-x))\, \rho\right) \tau n + \left(\frac{\hbar^2}{2\, m} + e^{-p4\, \rho} (p3 + p5\, x)\, \rho\right) \tau p$$

The kinetic terms:

```
HkinAPR = (h^2 / 2 / m + (n p3 + nn p5) Exp[-p4 n]) τn + (h^2 / 2 / m + (p3 n + np p5) Exp[-p4 n]) τp
```

$$\left(\frac{h^2}{2\, m} + e^{-n\, p4} (n\, p3 + nn\, p5)\right) \tau n + \left(\frac{h^2}{2\, m} + e^{-n\, p4} (n\, p3 + np\, p5)\right) \tau p$$

Pethick, et. al.'s definition of the P's and Q's:

```
P1 = (p3 / 2 - p5) Exp[-n p4]
```

$$e^{-n\, p4} \left(\frac{p3}{2} - p5\right)$$

```
P2 = (p3 / 2 + p5) Exp[-n p4]
```

$$e^{-n\, p4} \left(\frac{p3}{2} + p5\right)$$

```
Q1 = P1 / 2
```

$$\frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} - p5\right)$$

```
Q2 = P2 / 2
```

$$\frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} + p5\right)$$

Demonstrate that this gives us what we expect, namely, the kinetic part of the APR Hamiltonian:

```
HkinPRL = (h^2 / 2 / m + (P1 + P2) n - (Q1 - Q2) nn) τn + (h^2 / 2 / m + (P1 + P2) n - (Q1 - Q2) np) τp
```

$$\left(\frac{h^2}{2\, m} - nn \left(\frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} - p5\right) - \frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} + p5\right)\right) + n \left(e^{-n\, p4} \left(\frac{p3}{2} - p5\right) + e^{-n\, p4} \left(\frac{p3}{2} + p5\right)\right)\right) \tau n + \left(\frac{h^2}{2\, m} - np \left(\frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} - p5\right) - \frac{1}{2} e^{-n\, p4} \left(\frac{p3}{2} + p5\right)\right) + n \left(e^{-n\, p4} \left(\frac{p3}{2} - p5\right) + e^{-n\, p4} \left(\frac{p3}{2} + p5\right)\right)\right) \tau p$$

```
Simplify[HkinAPR - HkinPRL]
```

0

Now define the new terms P1f and P2f, and dQ2dn:

```
P1f = (Integrate[P1, {n, 0, np}] / np) /. np → n
```

$$\frac{e^{-n\,p4}\,(-1+e^{n\,p4})\,(p3-2\,p5)}{2\,n\,p4}$$

```
P2f = (Integrate[P2, {n, 0, np}] / np) /. np → n
```

$$\frac{e^{-n\,p4}\,(-1+e^{n\,p4})\,(p3+2\,p5)}{2\,n\,p4}$$

```
dQ2dn = D[Q2, n]
```

$$-\frac{1}{2}\,e^{-n\,p4}\,p4\left(\frac{p3}{2}+p5\right)$$

Write the gradient part of the Hamiltonian

```
Hgradient =
 -1 / 4 Simplify[(2 P1 + P1f - P2f) ] (nn[z] + np[z]) (nn''[z] + np''[z])
+1 / 2 Simplify[(Q1 + Q2)] (nn[z] nn''[z] + np[z] np''[z])
-1 / 4 Simplify[(Q1 - Q2)] (nn'[z]^2 + np'[z]^2)
+1 Simplify[dQ2dn] / 2 (nn[z] nn'[z] + np[z] np'[z]) (nn'[z] + np'[z])
```

$$-\frac{e^{-n\,p4}\,(n\,p4\,(p3-2\,p5)-2\,(-1+e^{n\,p4})\,p5)\,(nn[z]+np[z])\,(nn''[z]+np''[z])}{4\,n\,p4}$$

$$\frac{1}{4}\,e^{-n\,p4}\,p3\,(nn[z]\,nn''[z]+np[z]\,np''[z])$$

$$\frac{1}{4}\,e^{-n\,p4}\,p5\,(nn'[z]^2+np'[z]^2)$$

$$-\frac{1}{8}\,e^{-n\,p4}\,p4\,(p3+2\,p5)\,(nn'[z]+np'[z])\,(nn[z]\,nn'[z]+np[z]\,np'[z])$$

Use Paul’s rewriting to remove the second derivatives:

```
Hgradnew = Simplify[
  1 / 4 (3 P1 + 2 n D[P1, n] - P2) (nn'[z] + np'[z])^2 - 1 / 4 (3 Q1 + Q2) (nn'[z]^2 + np'[z]^2) -
   1 / 2 D[Q1, n] (nn[z] nn'[z]^2 + np[z] np'[z]^2 + (nn[z] + np[z]) nn'[z] np'[z])]
```

$$\frac{1}{8}\,e^{-n\,p4}\,\big((-2\,n\,p4\,(p3-2\,p5)-6\,p5+p4\,(p3-2\,p5)\,nn[z])\,nn'[z]^2+$$
$$(4\,(p3-n\,p3\,p4-4\,p5+2\,n\,p4\,p5)+p4\,(p3-2\,p5)\,nn[z]+p4\,(p3-2\,p5)\,np[z])\,nn'[z]\,np'[z]+$$
$$(-2\,n\,p4\,(p3-2\,p5)-6\,p5+p4\,(p3-2\,p5)\,np[z])\,np'[z]^2\big)$$

Put in the usual form, with density dependent Q’s

```
Qnnnew = 2 Simplify[(Hgradnew /. np'[z] → 0 /. n → nn[z] + np[z]) / nn'[z]^2]
```

$$\frac{1}{4}\,e^{-p4\,(nn[z]+np[z])}\,(-6\,p5-p4\,(p3-2\,p5)\,nn[z]-2\,p4\,(p3-2\,p5)\,np[z])$$

```
tmp = SeriesCoefficient[Series[Hgradnew, {nn′[z], 0, 1}], 1] / np′[z]
```

$$\frac{1}{8} e^{-n\, p4} \left(4 \left(p3 - n\, p3\, p4 - 4\, p5 + 2\, n\, p4\, p5\right) + p4 \left(p3 - 2\, p5\right) nn[z] + p4 \left(p3 - 2\, p5\right) np[z]\right)$$

```
Qnpnew = Simplify[(tmp /. n → nn[z] + np[z])]
```

$$\frac{1}{8} e^{-p4\,(nn[z]+np[z])} \left(4 \left(p3 - 4\, p5\right) - 3\, p4 \left(p3 - 2\, p5\right) nn[z] - 3\, p4 \left(p3 - 2\, p5\right) np[z]\right)$$

```
Qppnew = 2 Simplify[(Hgradnew /. nn′[z] → 0 /. n → nn[z] + np[z]) / np′[z]^2]
```

$$\frac{1}{4} e^{-p4\,(nn[z]+np[z])} \left(-6\, p5 - 2\, p4 \left(p3 - 2\, p5\right) nn[z] - p4 \left(p3 - 2\, p5\right) np[z]\right)$$

```
Qnnnew /. nn[z] → 0 /. np[z] → 0 /. p5 → -59.0
```

88.5

```
Qnpnew /. nn[z] → 0 /. np[z] → 0 /. p5 → -59.0 /. p3 → 89.9
```

162.95

Take derivatives so that we can easily write the Diff Eq's:

```
{Simplify[D[Qnnnew, nn[z]]], Simplify[D[Qnnnew, np[z]]]}
```

$$\left\{\frac{1}{4} e^{-p4\,(nn[z]+np[z])}\, p4 \left(-p3 + 8\, p5 + p4 \left(p3 - 2\, p5\right) nn[z] + 2\, p4 \left(p3 - 2\, p5\right) np[z]\right),\right.$$
$$\left.\frac{1}{4} e^{-p4\,(nn[z]+np[z])}\, p4 \left(p4 \left(p3 - 2\, p5\right) nn[z] + 2 \left(-p3 + 5\, p5 + p4 \left(p3 - 2\, p5\right) np[z]\right)\right)\right\}$$

```
{Simplify[D[Qnpnew, nn[z]]], Simplify[D[Qnpnew, np[z]]]}
```

$$\left\{\frac{1}{8} e^{-p4\,(nn[z]+np[z])}\, p4 \left(-7\, p3 + 22\, p5 + 3\, p4 \left(p3 - 2\, p5\right) nn[z] + 3\, p4 \left(p3 - 2\, p5\right) np[z]\right),\right.$$
$$\left.\frac{1}{8} e^{-p4\,(nn[z]+np[z])}\, p4 \left(-7\, p3 + 22\, p5 + 3\, p4 \left(p3 - 2\, p5\right) nn[z] + 3\, p4 \left(p3 - 2\, p5\right) np[z]\right)\right\}$$

```
{Simplify[D[Qppnew, nn[z]]], Simplify[D[Qppnew, np[z]]]}
```

$$\left\{\frac{1}{4} e^{-p4\,(nn[z]+np[z])}\, p4 \left(-2 \left(p3 - 5\, p5\right) + 2\, p4 \left(p3 - 2\, p5\right) nn[z] + p4 \left(p3 - 2\, p5\right) np[z]\right),\right.$$
$$\left.\frac{1}{4} e^{-p4\,(nn[z]+np[z])}\, p4 \left(-p3 + 8\, p5 + 2\, p4 \left(p3 - 2\, p5\right) nn[z] + p4 \left(p3 - 2\, p5\right) np[z]\right)\right\}$$

Check:

```
Simplify[
 Qnnnew nn′[z]^2 / 2 + Qnpnew np′[z] nn′[z] + Qppnew np′[z]^2 / 2 - Hgradnew /. n → nn[z] + np[z]]
```

0

Get boundary conditions:

```
nn = nn0 - δ Exp[-α x]
```

$$nn0 - e^{-x\,\alpha}\, \delta$$

```
np = np0 - ϵ Exp[-α x]
```

$np0 - e^{-x\alpha}\,\epsilon$

```
nnp = D[nn, x]; npp = D[np, x]; nnpp = D[nnp, x]; nppp = D[npp, x];
```

Old boundary condition equation:

```
eq1 =
 Simplify[Exp[α x] (Qnn nnpp + Qnp nppp) == Exp[α x] (dmundnn (nn - nn0) + dmundnp (np - np0))]
```

$\mathrm{dmundnn}\,\delta + \mathrm{dmundnp}\,\epsilon == \alpha^2\,(\mathrm{Qnn}\,\delta + \mathrm{Qnp}\,\epsilon)$

```
Solve[eq1, ϵ]
```

$\left\{\left\{\epsilon \to \frac{-\mathrm{dmundnn}\,\delta + \mathrm{Qnn}\,\alpha^2\,\delta}{\mathrm{dmundnp} - \mathrm{Qnp}\,\alpha^2}\right\}\right\}$

New boundary condition equation:

```
eq2 = (Qnn nnpp + Qnp nppp) ==
   (dmundnn (nn - nn0) + dmundnp (np - np0) + dqnndnn nnp^2 / 2 + dqnpdnn nnp npp + dqppdnn npp^2 / 2)
```

$$-e^{-x\alpha}\,\mathrm{Qnn}\,\alpha^2\,\delta - e^{-x\alpha}\,\mathrm{Qnp}\,\alpha^2\,\epsilon == -\mathrm{dmundnn}\,e^{-x\alpha}\,\delta + \frac{1}{2}\,\mathrm{dqnndnn}\,e^{-2x\alpha}\,\alpha^2\,\delta^2 - \mathrm{dmundnp}\,e^{-x\alpha}\,\epsilon + \mathrm{dqnpdnn}\,e^{-2x\alpha}\,\alpha^2\,\delta\,\epsilon + \frac{1}{2}\,\mathrm{dqppdnn}\,e^{-2x\alpha}\,\alpha^2\,\epsilon^2$$

Note that since the exponents are even smaller, that we can use the old
boundary conditions to first order.

Now go back to the Skyrme form:

```
Clear["nn"]; Clear["np"]; Clear["n"];
```

```
P1 = t1[n] / 4; P2 = t2[n] / 4; Q1 = t1[n] / 8; Q2 = t2[n] / 8;
```

```
Hgradnew = Simplify[
  1 / 4 (3 P1 + 2 n D[P1, n] - P2) (nn'[z] + np'[z])^2 - 1 / 4 (3 Q1 + Q2) (nn'[z]^2 + np'[z]^2) -
   1 / 2 D[Q1, n] (nn[z] nn'[z]^2 + np[z] np'[z]^2 + (nn[z] + np[z]) nn'[z] np'[z])]
```

$$\frac{1}{32}\,(3\,t1[n]\,(nn'[z]^2 + 4\,nn'[z]\,np'[z] + np'[z]^2) - t2[n]\,(3\,nn'[z]^2 + 4\,nn'[z]\,np'[z] + 3\,np'[z]^2) + 2\,(nn'[z] + np'[z])\,((2\,n - nn[z])\,nn'[z] + (2\,n - np[z])\,np'[z])\,t1'[n])$$

```
Qnnnew = 2 Simplify[(Hgradnew /. np'[z] → 0) / nn'[z]^2]
```

$\frac{1}{16}\,(3\,t1[n] - 3\,t2[n] + 2\,(2\,n - nn[z])\,t1'[n])$

```
tmp = SeriesCoefficient[Series[Hgradnew, {nn'[z], 0, 1}], 1] / np'[z]
```

$$\frac{1}{np'[z]}\left(\frac{3}{8}\,t1[n]\,np'[z] - \frac{1}{8}\,t2[n]\,np'[z] + \frac{1}{4}\,n\,np'[z]\,t1'[n] - \frac{1}{16}\,nn[z]\,np'[z]\,t1'[n] - \frac{1}{16}\,np[z]\,np'[z]\,t1'[n]\right)$$

```
Qnpnew = Simplify[(tmp)]
```

$\frac{1}{16}\,(6\,t1[n] - 2\,t2[n] - (-4\,n + nn[z] + np[z])\,t1'[n])$

```
Qppnew = 2 Simplify[(Hgradnew /. nn′[z] → 0) / np′[z]^2]
```

$$\frac{1}{16}\left(3\,\mathrm{t1}[n]-3\,\mathrm{t2}[n]+2\,(2\,n-\mathrm{np}[z])\,\mathrm{t1}'[n]\right)$$

Now find t1 and t2 as a function of the effective masses

```
tx1 = 1 / 4 (t1 + t2); tx2 = 1 / 4 (t2 / 2 - t1 / 2);
```

```
eq1 = msn == mn / (1 + 2 (n tx1 + nn tx2) mn)
```

$$\mathrm{msn} == \frac{\mathrm{mn}}{1+2\,\mathrm{mn}\left(\frac{1}{4}\,\mathrm{nn}\left(-\frac{\mathrm{t1}}{2}+\frac{\mathrm{t2}}{2}\right)+\frac{1}{4}\,n\,(\mathrm{t1}+\mathrm{t2})\right)}$$

```
eq2 = msp == mp / (1 + 2 (n tx1 + np tx2) mp)
```

$$\mathrm{msp} == \frac{\mathrm{mp}}{1+2\,\mathrm{mp}\left(\frac{1}{4}\,\mathrm{np}\left(-\frac{\mathrm{t1}}{2}+\frac{\mathrm{t2}}{2}\right)+\frac{1}{4}\,n\,(\mathrm{t1}+\mathrm{t2})\right)}$$

```
Simplify[Solve[{eq1, eq2}, {t1, t2}]]
```

$$\Big\{\Big\{\mathrm{t1}\to(2\,\mathrm{mn\,mp\,msn}\,n-2\,\mathrm{mn\,mp\,msp}\,n-2\,\mathrm{mn\,msn\,msp}\,n+2\,\mathrm{mp\,msn\,msp}\,n+\mathrm{mn\,mp\,msn\,nn}-\mathrm{mn\,msn\,msp\,nn}-\mathrm{mn\,mp\,msp\,np}+\mathrm{mp\,msn\,msp\,np})\,/\,(\mathrm{mn\,mp\,msn\,msp}\,n\,\mathrm{nn}-\mathrm{mn\,mp\,msn\,msp}\,n\,\mathrm{np}),$$
$$\mathrm{t2}\to-\frac{-(-4\,\mathrm{mp}+4\,\mathrm{msp})\,(2\,\mathrm{mn\,msn}\,n-\mathrm{mn\,msn\,nn})+(-4\,\mathrm{mn}+4\,\mathrm{msn})\,(2\,\mathrm{mp\,msp}\,n-\mathrm{mp\,msp\,np})}{4\,\mathrm{mn\,mp\,msn\,msp}\,n\,\mathrm{nn}-4\,\mathrm{mn\,mp\,msn\,msp}\,n\,\mathrm{np}}\Big\}\Big\}$$