

O₂scl_part - Particles and Nuclei Sub-Library for O₂scl

Version 0.910

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1.1 Feature Overview

O₂scl_part is sub-library for working with particles and nuclei designed to work with O₂scl . It includes

- Several classes for computing the thermodynamic properties of interacting and non-interacting fermions, bosons and nuclei
- Currently available experimental nuclear mass data
- Theoretical nuclear mass models and mass formulas
- Containers holding distributions of nuclei

1.2 Quick Reference to User's Guide

- [Particles](#)
- [Atomic Nuclei](#)
- [Example Source Code](#)
- [Bibliography](#)
- [Todo List](#)
- [Ideas for Future Development](#)

2 Particles

These classes in the library O₂scl_part calculate the thermodynamic properties of interacting and non-interacting quantum and classical particles.

The class [part](#) is the basic structure for a particle:

- [part::m](#) - mass
- [part::g](#) - degeneracy factor (e.g. $2j + 1$)
- [part::n](#) - number density
- [part::ed](#) - energy density
- [part::pr](#) - pressure
- [part::en](#) - entropy density
- [part::ms](#) - effective mass, i.e. m^*
- [part::nu](#) - effective chemical potential, i.e. ν
- [part::inc_rest_mass](#) - True if the rest mass is included
- [part::non_interacting](#) - False if the particle includes interactions

The data members `part::ms` and `part::nu` allow one to specify modifications to the mass and the chemical potential due to interactions. This allows one to calculate the properties of particle due to interactions so long as the basic form of the free-particle dispersion relation is unchanged, i.e.

$$\sqrt{k^2 + m^2} - \mu \rightarrow \sqrt{k^2 + m^{*2}} - \nu$$

If the particle is non-interacting, then `part::nu` and `part::ms` are sometimes used by `O2scl_part` functions for temporary storage.

If `part::inc_rest_mass` is `true` (as is the default in all of the classes except `nucleus`), then all functions include the rest mass energy density in the energy density, the "mu" functions expect that the rest mass is included in `part::mu` or `part::nu` as input and the "density" functions output `part::mu` or `part::nu` including the rest mass.

When `part::inc_rest_mass` is true, antiparticles are implemented by choosing the antiparticle chemical potential to be $-\mu$, and when `inc_rest_mass` is false, antiparticles are implemented by choosing the chemical potential of the antiparticles to be $-\mu - 2m$.

The thermodynamic identity used to compute the pressure for interacting particles is

$$P = -\epsilon + sT + \nu n$$

where `part::nu` is used. This way, the particle class doesn't need to know about the structure of the interactions to ensure that the thermodynamic identity is satisfied. Note that in the `O2scl_eos` library, where in the equations of state the normal thermodynamic identity is used

$$P = -\epsilon + sT + \mu n$$

Frequently, the interactions which create an effective chemical potential which is different than `part::mu` thus create extra terms in the pressure and the energy density for the given equation of state.

At zero temperature, fermions and bosons can be treated exactly in the classes `fermion` and `boson`. The `quark` class is a descendant of the `fermion` class which contains extra data members for the quark condensate and the contribution to the bag constant. The `classical` class is a descendant of both `fermion` and `boson` and calculates everything in the classical limit.

At finite temperature, there are different classes corresponding to different approaches to computing the integrals over the distribution functions. The approximation scheme from [Johns96](#) is used in `eff_boson`, `eff_fermion`, and `eff_quark`. An exact method employing direct integration of the distribution functions is used in `rel_boson` and `rel_fermion`, but these are necessarily quite a bit slower.

The class `eff_fermion` usually works to within about 1 part in 10^4 , but can be as bad as 1 part in 10^2 in some more extreme cases. The default settings for `rel_fermion` give an accuracy of at least 1 part in 10^6 (and frequently better than this). For `rel_fermion`, some additional accuracy may be obtained by decreasing the integration tolerances.

The class `nonrel_fermion` assumes a non-relativistic dispersion relation for fermions. It includes zero-temperature methods and an exact method for finite temperatures. The non-relativistic integrands are much simpler and `nonrel_fermion` uses the appropriate GSL functions (which are nearly exact) to compute them.

Units:

Factors of \hbar, c and k_B have been removed everywhere, so that mass, energy, and temperature all have the same units. Number and entropy densities have units of mass cubed (or energy cubed). This particle classes can be used with any system of units which is based on powers of one unit, i.e. $[n] = [T]^3 = [m]^3 = [P]^{3/4} = [\epsilon]^{3/4}$, etc.

Derivative information:

Sometimes it is useful to know derivatives like ds/dT in addition to the energy and pressure. There are three classes which compute these derivatives for fermions and classical particles. The class `sn_classical` handles the nondegenerate limit, `sn_fermion` handles fermions and `sn_nr_fermion` handles nonrelativistic fermions. These classes compute the derivatives

$$\left(\frac{dn}{d\mu}\right)_T, \quad \left(\frac{dn}{dT}\right)_\mu, \quad \text{and} \quad \left(\frac{ds}{dT}\right)_\mu.$$

All other first derivatives of the thermodynamic functions can be written in terms of these three. To see how to compute the specific heat, for example, see the discussion in the documentation of `part_deriv`.

3 Atomic Nuclei

Nuclei

Atomic nuclei, class [nucleus](#), are implemented as descendants of [classical](#). This class sets the value of [nucleus::inc_rest_mass](#) to false by default.

Nuclear mass formulas are given as children of [nuclear_mass](#). The class [ame_mass](#) provides the experimental data from [Audi95](#) or [Audi03](#), the class [mnmsk_mass](#) provides the mass formula from [Moller95](#), and the class [hfb_mass](#) provides the mass formula from [Goriely02](#), [Samyn04](#), or [Goriely07](#). A simple semi-empirical mass formula is given in [semi_empirical_mass](#) and this can be fit to experimentally measured masses using [mass_fit](#).

The class [nuclear_dist](#) provides an generic base class for a collection of several nuclei with an STL-like iterator. There are two implementations of this base class, [simple_dist](#) which provides a simple distribution and [full_dist](#) which enumerates all the nuclei for a given mass formula.

4 Example Source Code

4.1 Example list

- [Particle example](#)
- [Nuclear mass fit example](#)

4.2 Particle example

```
/* Example: ex_part.cpp
-----
*/

#include <cmath>
#include <o2scl/test_mgr.h>
#include <o2scl/constants.h>
#include <o2scl/eff_fermion.h>
#include <o2scl/rel_fermion.h>
#include <o2scl/classical.h>

using namespace std;
using namespace o2scl;
using namespace o2scl_const;

int main(void) {
    test_mgr t;
    t.set_output_level(1);

    // Compare the method from rel_fermion to the more approximate
    // scheme used in eff_fermion. We work in units of inverse Fermis,
    // so that energy density is fm-4. We also use a classical
    // particle, to compare to the nondegenerate approximation.
    eff_fermion eff;
    rel_fermion relf;
    classical cla;

    fermion e(o2scl_fm::mass_electron,2.0);
    fermion e2(o2scl_fm::mass_electron,2.0);
    fermion e3(o2scl_fm::mass_electron,2.0);

    // Compute the pressure at a density of 0.0001 fm-3 and a
    // temperature of 10 MeV. At these temperatures, the electrons are
    // non-degenerate, and Boltzmann statistics nearly applies.
    e.n=0.0001;
    eff.calc_density(e,10.0/hc_mev_fm);
```

```

e2.n=0.0001;
relf.calc_density(e2,10.0/hc_mev_fm);
e3.n=0.0001;
cla.calc_density(e3,10.0/hc_mev_fm);

cout << e.pr << " " << e2.pr << " " << e3.pr << " "
      << e.n*10.0/hc_mev_fm << endl;

// Test
t.test_rel(e.pr,e2.pr,1.0e-2,"EFF vs. exact");
t.test_rel(e2.pr,e3.pr,4.0e-1,"classical vs. exact");
t.test_rel(e.n*10.0/hc_mev_fm,e3.pr,1.0e-1,"classical vs. ideal gas law");

// Compute the pressure at a density of 0.1 fm-3 and a
// temperature of 1 MeV. At these temperatures, the electrons are
// strongly degenerate
e.n=0.0001;
eff.calc_density(e,10.0/hc_mev_fm);
e2.n=0.0001;
relf.calc_density(e2,10.0/hc_mev_fm);
cout << e.pr << " " << e2.pr << endl;

// Test
t.test_rel(e.pr,e2.pr,1.0e-2,"EFF vs. exact");

// Now add the contribution to the pressure from positrons using the
// implementation of part::pair_density()
e.n=0.0001;
eff.pair_density(e,10.0/hc_mev_fm);
e2.n=0.0001;
relf.pair_density(e2,10.0/hc_mev_fm);
cout << e.pr << " " << e2.pr << endl;

// Test
t.test_rel(e.pr,e2.pr,1.0e-2,"EFF vs. exact");

t.report();
return 0;
}
// End of example

```

4.3 Nuclear mass fit example

```

/* Example: ex_mass_fit.cpp
-----
*/

#include <iostream>
#include <o2scl/test_mgr.h>
#include <o2scl/mass_fit.h>
#ifdef O2SCL_HDF_IN_EXAMPLES
#include <o2scl/hdf_file.h>
#include <o2scl/hdf_nucmass_io.h>
#endif

using namespace std;
using namespace o2scl;
using namespace o2scl_const;
using namespace o2scl_fm;

int main(void) {
    test_mgr t;
    t.set_output_level(1);

    cout.setf(ios::scientific);

```

```

#ifdef O2SCL_HDF_IN_EXAMPLES

// The RMS deviation of the fit
double res;
// The mass formula to be fitted
semi_empirical_mass sem;
// The fitting class
mass_fit mf;

// Load the experimental data
ame_mass ame;
o2scl_hdf::ame_load(ame, "");
mf.set_exp_mass(ame);

// Perform the fit
mf.fit(sem, res);

// Output the results
cout << sem.B << " " << sem.Sv << " " << sem.Ss << " "
    << sem.Ec << " " << sem.Epair << endl;
cout << res << endl;
t.test_gen(res<4.0, "Successful fit.");

#else
    cout << "No fitting was performed because O2scl appears not to have been "
        << "compiled with HDF support." << endl;
#endif

    t.report();
    return 0;
}
// End of example

```

5 Bibliography

Some of the references which contain links should direct you to the work referred to directly through [dx.doi.org](https://doi.org/).

Audi95: [G. Audi](#) and [A. H. Wapstra](#), Nucl. Phys. A **595** (1995) 409-480.

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Goriely02: [S. Goriely](#), [M. Samyn](#), [P.-H. Heenen](#), [J. M. Pearson](#), and [F. Tondeur](#), Phys. Rev. C **66** (2002) 024326.

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Johns96: [Johns, P.J. Ellis](#), and [J.M. Lattimer](#), Astrophys. J. **473** (1996) 1020.

Landau: L. D. Landau, "Statistical Physics: Part 1", 3rd Edition.

Moller95: [P. Moller](#), [J.R. Nix](#), [W.D. Myers](#), and [W.J. Swiatecki](#), At. Data Nucl. Data Tables **59** (1995) 185.

Moller97: [P. Moller](#), [J.R. Nix](#), and [K.-L. Kratz](#) At. Data Nucl. Data Tables **66** (1997) 131.

Samyn04: [M. Samyn](#), [S. Goriely](#), [M. Bender](#) and [J. M. Pearson](#), Phys. Rev. C **70** (2004) 044309.

6 Ideas for Future Development

Class [ame_mass](#)

There are definitions of the atomic mass unit and other constants that are defined by the 1995 and 2003 atomic mass evaluations which are not used at present. These could be implemented by making a separate copy of the neutron and proton masses inside

this class.

Create a caching and more intelligent search system for the table. The table is sorted by A and then N, so we could probably just copy the search routine from `mnmsk_mass`, which is sorted by Z and then N.

Allow the user to select only masses which are not estimated.

Class `classical`

Write a `calc_density_zerot()` function for completeness?

Class `eff_fermion`

Use bracketing to speed up one-dimensional root finding.

Class `fermion_eval_thermo`

Create a Chebyshev approximation for inverting the the Fermi functions for `massless_calc_density()` functions?

Global `fermion_eval_thermo::calibrate` (`fermion &f, int verbose=0, std::string fname=""`)

Also calibrate massless fermions?

Convert into separate class?

Global `fermion_eval_thermo::massless_pair_density` (`fermion &f, double temper`)

This could be improved by including more terms in the expansions.

Class `fermion_zerot`

Use `hypot()` and other more accurate functions for the analytic expressions for the zero temperature integrals. [Progress has been made, but there are probably other functions which may break down for small but finite masses and temperatures]

Class `frdm_mass`

Add microscopic part.

Class `mass_fit`

Convert to a real fit with errors and covariance, etc.

Class `nonrel_fermion`

This could be improved by performing a Chebyshev approximation (for example) to invert the density integral so that we don't need to use a solver.

Class `nuclear_mass`

Find a way to include the electron binding energy contribution, possibly with the help of a theoretical model.

Class `nuclear_mass_fit`

This shouldn't be a child of `nuclear_mass_cont`?

Global `nuclear_mass_info::parse_elstring` (`std::string ela, int &Z, int &N, int &A`)

Warn about malformed combinations like Carbon-5

Right now, `n4` is interpreted incorrectly as Nitrogen-4, rather than the tetra-neutron.

Class `rel_fermion`

The expressions which appear in in the integrand functions `density_fun()`, etc. could likely be improved, especially in the case where `inc_rest_mass=false`. There should not be a need to check if `ret` is finite.

It appears this doesn't compute the uncertainty in the chemical potential or density with `calc_density()`. This could be fixed.

I'd like to change the lower limit on the entropy integration, but the value in the code at the moment (stored in `ll`) makes `bm_part2.cpp` worse.

`pair_mu()` should set the antiparticle integrators as done in `sn_fermion`.

Class `simple_dist`

Make the vector constructor into a template so it accepts any type. Do the same for `set_dist()`.

Class `sn_fermion`

It might be worth coding up direct differentiation, or differentiating the `eff` results, as these may succeed more generally.

This class will have difficulty with extremely degenerate or extremely non-degenerate systems. Fix this.

Create a more intelligent method for dealing with bad initial guesses for the chemical potential in `calc_density()`.

7 Todo List

Class `eff_boson`

- Better documentation (see `eff_fermion`)
- Remove the 'meth2' stuff
- Remove static variables `fix_density` and `stat_temper`
- Remove `exit()` calls

Class `eff_fermion`

There's still `def_err_hnd.set_mode(0)` in the testing code, probably because the solver has a hard time for extreme values.

Global `eff_fermion::calc_mu` (`fermion &f, double temper`)

Should see if the function actually works if $(\mu - m)/T = -199$.

Class `frdm_mass`

- Fix pairing energy and double vs. int
- Document `drip_binding_energy()`, etc.
- Decide on number of fit parameters (10 or 12?) or let the user decide
- Document the protected variables
- Set the neutron and proton masses and `hbarc` to Moller et al.'s values

Class `nonrel_fermion`

- Check behaviour of `calc_density()` at zero density, and compare with that from `eff_fermion`, `rel_fermion`, and `classical`.
- Implement `pair_density()` and `pair_mu()`.
- Make sure to test with non-interacting equal to true or false, and document whether or not it works with both `inc_rest_mass` equal to true or false

Class `rel_boson`

- Testing not completely finished.

Class `sn_classical`

- This does not work with `inc_rest_mass=true`

Class `sn_fermion`

- This needs to be corrected to calculate $\sqrt{k^2 + m^{*2}} - m$ gracefully when $m^* \approx m \ll k$.
- Call error handler if `inc_rest_mass` is true or update to properly treat the case when `inc_rest_mass` is true.

8 Data Structure Documentation

8.1 `ame_entry` Class Reference

Atomic mass entry structure.

```
#include <nuclear_mass.h>
```

8.1.1 Detailed Description

Atomic mass entry data object for `ame_mass`.

In cases where the decimal point in the original table was replaced with a #, the associated accuracy field is set to `estimated`. In cases where the original table contained an asterisk to indicate a value was not calculable, the accuracy field is set to `not_calculable` and the value is zero. If `O2scl` internally computed the value instead of reading it from the original table, the accuracy field is set to `intl_computed`. In cases where either `orig` or `bdmode` in the original table was blank, the string is set to "blank".

Binding energies are defined with a positive sign, so that lead has a binding energy of +8 MeV.

Definition at line 457 of file nuclear_mass.h.

Data Fields

- int [NMZ](#)
N-Z.
- int [N](#)
Neutron number.
- int [Z](#)
Proton number.
- int [A](#)
Atomic number.
- char [el](#) [4]
Element name.
- char [orig](#) [5]
Data origin.
- double [mass](#)
Mass excess (in keV)
- double [dmass](#)
Mass excess uncertainty (in keV)
- int [mass_acc](#)
Mass accuracy flag.
- double [be](#)
Binding energy (in keV, given in the '95 data)
- double [dbe](#)
Binding energy uncertainty (in keV, given in the '95 data)
- int [be_acc](#)
Binding energy accuracy flag.
- double [beoa](#)
Binding energy / A (in keV, given in the '03 data)
- double [dbeoa](#)
Binding energy / A uncertainty (in keV, given in the '03 data)
- int [beoa_acc](#)
Binding energy / A accuracy flag.
- char [bdmode](#) [3]
Beta decay mode.
- double [bde](#)
Beta-decay energy (in keV)
- double [dbde](#)
Beta-decay energy uncertainty (in keV)
- int [bde_acc](#)
Beta-decay energy accuracy flag.
- int [A2](#)
?
- double [amass](#)
Atomic mass (in keV)
- double [damass](#)
Atomic mass uncertainty (in keV)
- int [amass_acc](#)
Atomic mass accuracy flag.

Static Public Attributes

Accuracy modes

- static const int [measured](#) = 0
Measured value from source data.

- static const int `estimated` = 1
Value estimated in source data.
- static const int `not_calculable` = 2
Value listed in data as not calculable.
- static const int `intl_computed` = 3
Value computed by O_2scl .

The documentation for this class was generated from the following file:

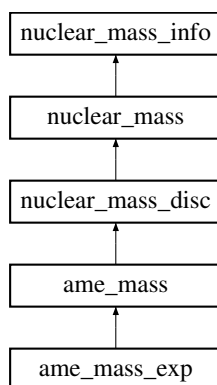
- `nuclear_mass.h`

8.2 ame_mass Class Reference

Mass formula from the Atomic Mass Evaluation (2005 and 1993)

```
#include <nuclear_mass.h>
```

Inheritance diagram for `ame_mass`:



8.2.1 Detailed Description

Note

This class requires data stored in an HDF file and thus requires HDF support for normal usage.

This class provides an interface to the atomic mass table using data from [Audi95](#) and [Audi03](#).

There are four data sets, selected by the specification of the `version` string in the constructor.

- "95rmd" - "Recommended" data from [Audi95](#) (`ame95rmd.o2`)
- "95exp" - "Experimental" data from [Audi95](#) (`ame95exp.o2`)
- "03round" - "Rounded" data from [Audi03](#) (`ame03round.o2`)
- "03" - Data from [Audi03](#) (default) (`ame03.o2`)

If any string other than these four is used, the default data is loaded. If the constructor cannot find the data file (e.g. because of a broken installation), then `ame::is_loaded()` returns false.

The 1995 data provided the binding energy stored in `ame_entry::be` and `ame_entry::dbe`, while the 2003 data provided the binding energy divided by the atomic number stored in `ame_entry::beoa` and `ame_entry::dbeoa`. When the 1995 data is used `ame_entry::beoa` and `ame_entry::dbeoa` are calculated automatically, and when the 2003 data is used `ame_entry::be` and `ame_entry::dbe` are

calculated automatically. To indicate that O₂scl has automatically calculated a value in this way, the associated accuracy field is set to [ame_entry::intl_computed](#).

Note that all uncertainties are 1 sigma uncertainties.

The functions [mass_excess\(\)](#) and [nuclear_mass::mass_excess_d\(\)](#) directly return the value from the Audi et al. data. For consistency, the functions [nuclear_mass::binding_energy\(\)](#), [nuclear_mass::binding_energy_d\(\)](#), [nuclear_mass::total_mass\(\)](#), and [nuclear_mass::total_mass_d\(\)](#) return values which are automatically computed with the O₂scl values for the neutron and proton mass in **o2scl_fm** (which are obtained from CODATA). In order to obtain the value of the binding energy as reported in the original Audi et al. data instead of the value computed from the mass excess, you can use the function [get_ZN\(\)](#), and access the corresponding entry from the Audi et al. data directly.

See also the documentation for the class structure for each table entry in [ame_entry](#).

Idea for Future There are definitions of the atomic mass unit and other constants that are defined by the 1995 and 2003 atomic mass evaluations which are not used at present. These could be implemented by making a separate copy of the neutron and proton masses inside this class.

Idea for Future Create a caching and more intelligent search system for the table. The table is sorted by A and then N, so we could probably just copy the search routine from [mmmsk_mass](#), which is sorted by Z and then N.

Idea for Future Allow the user to select only masses which are not estimated.

Definition at line 609 of file [nuclear_mass.h](#).

Public Member Functions

- [ame_mass](#) ()
Create a collection specified by version.
- virtual const char * [type](#) ()
Return the type, "ame_mass".
- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double [mass_excess](#) (int Z, int N)
Given Z and N, return the mass excess in MeV.
- [ame_entry](#) [get_ZN](#) (int l_Z, int l_N)
Get element with Z=l_Z and N=l_N (e.g. 82,126).
- [ame_entry](#) [get_ZA](#) (int l_Z, int l_A)
Get element with Z=l_Z and A=l_A (e.g. 82,208).
- [ame_entry](#) [get_elA](#) (std::string l_el, int l_A)
Get element with name l_el and A=l_A (e.g. "Pb",208).
- [ame_entry](#) [get](#) (std::string [nucleus](#))
Get element with string (e.g. "Pb208")
- bool [is_loaded](#) ()
Returns true if data has been loaded.
- int [set_data](#) (int n_mass, [ame_entry](#) *m, std::string ref)
Set data.
- int [get_nentries](#) ()
Return number of entries.

Protected Attributes

- int [n](#)
The number of entries (about 3000).
- std::string [reference](#)
The reference for the original data.
- [ame_entry](#) * [mass](#)
The array containing the mass data of length ame::n.

8.2.2 Member Function Documentation

8.2.2.1 int ame_mass::set_data (int *n_mass*, ame_entry * *m*, std::string *ref*)

This function is used by the HDF I/O routines.

The documentation for this class was generated from the following file:

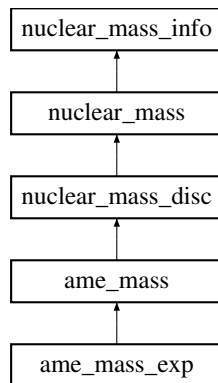
- nuclear_mass.h

8.3 ame_mass_exp Class Reference

The experimental values from Moller, Nix, Myers and Swiatecki.

```
#include <nuclear_mass.h>
```

Inheritance diagram for ame_mass_exp:



8.3.1 Detailed Description

Note

This class requires data stored in an HDF file and thus requires HDF support for normal usage.

Definition at line 674 of file nuclear_mass.h.

Public Member Functions

- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.

The documentation for this class was generated from the following file:

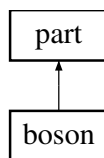
- nuclear_mass.h

8.4 boson Class Reference

Boson class.

```
#include <boson.h>
```

Inheritance diagram for boson:



8.4.1 Detailed Description

For bosons:

- if either ν or μ is greater than m , then they are taken to be equal to m
- All contributions from any type of condensate are ignored.

This Mathematica notebook contains the series expansions for the bosonic integrals. functions.

```
doc/o2scl/extras/boson.nb
doc/o2scl/extras/boson.pdf
```

Definition at line 66 of file boson.h.

Public Member Functions

- `boson` (double $m=0.0$, double $g=0.0$)
Create a boson with mass m and degeneracy g .
- virtual void `massless_calc_mu` (double $temper$)
Calculate properties of massless bosons.
- virtual const char * `type` ()
Return string denoting type ("boson")

Data Fields

- double `co`
The condensate.

8.4.2 Member Function Documentation

8.4.2.1 virtual void boson::massless_calc_mu (double *temper*) [virtual]

The expressions used are exact. The chemical potentials are ignored and the scalar density is set to zero

8.4.3 Field Documentation

8.4.3.1 double boson::co

The condensate variable is mostly ignored by class boson and its descendants, and is provided for user storage.

Definition at line 75 of file boson.h.

The documentation for this class was generated from the following file:

- boson.h

8.5 classical Class Reference

Classical particle class.

```
#include <classical.h>
```

8.5.1 Detailed Description

Idea for Future Write a `calc_density_zerot()` function for completeness?

Definition at line 44 of file `classical.h`.

Public Member Functions

- `classical ()`
Create a classical particle with mass m and degeneracy g .
- virtual void `calc_mu (part &p, double temper)`
Calculate properties as function of chemical potential.
- virtual void `calc_density (part &p, double temper)`
Calculate properties as function of density.
- virtual const char * `type ()`
Return string denoting type ("classical")

The documentation for this class was generated from the following file:

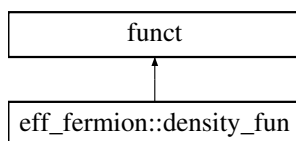
- `classical.h`

8.6 eff_fermion::density_fun Class Reference

Define the function which solves for the chemical potential given the density [protected subclass of `eff_fermion`].

```
#include <eff_fermion.h>
```

Inheritance diagram for `eff_fermion::density_fun`:



8.6.1 Detailed Description

Definition at line 227 of file `eff_fermion.h`.

Public Member Functions

- `density_fun (eff_fermion &ef, fermion &f, double T)`
- double `operator() (double x)`
Fix density for `eff_fermion::calc_density()`

Protected Attributes

- [eff_fermion](#) & [ef_](#)
- [fermion](#) & [f_](#)
- double [T_](#)

The documentation for this class was generated from the following file:

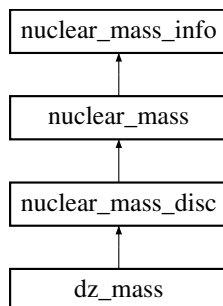
- [eff_fermion.h](#)

8.7 dz_mass Class Reference

Duflo-Zuker mass formula.

```
#include <nuclear_mass.h>
```

Inheritance diagram for dz_mass:



8.7.1 Detailed Description

Definition at line 1129 of file nuclear_mass.h.

Public Member Functions

- [dz_mass](#) (std::string model="96", bool external=false)
Create a new mass formula object.
- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double [mass_excess](#) (int Z, int N)
Given Z and N, return the mass excess in MeV.
- bool [is_loaded](#) ()
Verify that the constructor properly loaded the table.
- virtual const char * [type](#) ()
Return the type, "dz_mass".
- int [get_nentries](#) ()
Return number of entries.

Protected Attributes

- std::string [reference](#)
The reference for the original data.
- table [data](#)
Table containing the data.

- int [last](#)
The last table index for caching.
- int [n](#)
The total number of entries.

The documentation for this class was generated from the following file:

- nuclear_mass.h

8.8 eff_boson Class Reference

Boson class from fitting method.

```
#include <eff_boson.h>
```

8.8.1 Detailed Description

Todo Better documentation (see [eff_fermion](#))

Remove the 'meth2' stuff

Remove static variables fix_density and stat_temper

Remove exit() calls

Definition at line 52 of file eff_boson.h.

Public Member Functions

- [eff_boson](#) ()
Create a boson with mass m and degeneracy g .
- int [load_coefficients](#) (int ctype)
Load coefficients for finite-temperature approximation.
- virtual void [calc_mu](#) (boson &b, double temper)
Calculate thermodynamic properties as function of chemical potential.
- virtual void [calc_density](#) (boson &b, double temper)
Calculate thermodynamic properties as function of density.
- virtual void [pair_mu](#) (boson &b, double temper)
Calculate thermodynamic properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) (boson &b, double temper)
Calculate thermodynamic properties with antiparticles as function of density.
- void [set_psi_root](#) (root< [funct](#) > &rp)
Set the solver for use in calculating ψ .
- void [set_density_mroot](#) (mroot< [mm_funct](#)<> > &rp)
Set the solver for use in calculating the chemical potential from the density.
- void [set_meth2_root](#) (root< [funct](#) > &rp)
Set the solver for use in calculating the chemical potential from the density (meth2=true)
- virtual const char * [type](#) ()

Data Fields

- [gsl_mroot_hybrids](#)< [mm_funct](#)<> > [def_density_mroot](#)
The default solver for [calc_density\(\)](#) and [pair_density\(\)](#)
- [cern_mroot_root](#)< [funct](#) > [def_psi_root](#)
The default solver for ψ .
- [cern_mroot_root](#)< [funct](#) > [def_meth2_root](#)
The default solver for [calc_density\(\)](#) and [pair_density\(\)](#)

Static Public Attributes

- static const int `cf_boselat3` = 1
A set of coefficients from Jim Lattimer.
- static const int `cf_bosejel21` = 2
A set of coefficients from Johns96.
- static const int `cf_bosejel22` = 3
A set of coefficients from Johns96.
- static const int `cf_bosejel34` = 4
A set of coefficients from Johns96.
- static const int `cf_bosejel34cons` = 5
The set of coefficients from Johns96 which retains better thermodynamic consistency.

Protected Member Functions

- double `solve_fun` (double x, double &psi)
The function which solves for h from ψ .
- int `density_fun` (size_t nv, const `ovector_base` &x, `ovector_base` &y)
Fix density for `calc_density()`
- int `pair_density_fun` (size_t nv, const `ovector_base` &x, `ovector_base` &y)
Fix density for `pair_density()`

Protected Attributes

- `umatrix` `Pmnb`
The coefficients.
- int `sizem`
The number of coefficient rows.
- int `sizen`
The number of coefficient columns.
- double `parma`
The parameter, a.
- double `fix_density`
Temporary storage.
- `boson` * `bp`
Desc.
- double `T`
Desc.
- `mroot`< `mm_funct`<> > * `density_mroot`
The solver for `calc_density()`
- `root`< `funct` > * `psi_root`
The solver to compute h from ψ .
- `root`< `funct` > * `meth2_root`
The solver for `calc_density()`

8.8.2 Member Function Documentation

8.8.2.1 int eff_boson::load_coefficients (int ctype)

Presently acceptable values of `fn` are: `boselat3` from Lattimer's notes `bosejel21`, `bosejel22`, `bosejel34`, and `bosejel34cons` from Johns96.

The documentation for this class was generated from the following file:

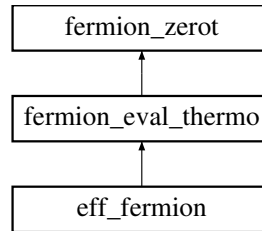
- `eff_boson.h`

8.9 eff_fermion Class Reference

Fermion class from fitting method.

```
#include <eff_fermion.h>
```

Inheritance diagram for eff_fermion:



8.9.1 Detailed Description

Based on the fitting method of [Johns96](#) which is an update of the method from [Eggleton73](#). This method is approximate, but very fast. For a more accurate (but slower) method, use [rel_fermion](#).

Given the chemical potential and the temperature the functions [calc_mu\(\)](#) and [pair_mu\(\)](#) work by solving the equation (c.f. Eq. 15 in [Johns96](#))

$$\psi = 2\sqrt{1 + f/a} + \log \left(\frac{\sqrt{1 + f/a} - 1}{\sqrt{1 + f/a} + 1} \right)$$

for f given $\psi = (\mu - m)/T$. If $f/a < 10^{-10}$, then the alternative expression

$$\psi = 2(1 + f/(2a)) + \log \left[\frac{f/(2a)}{(1 + f/(2a))} \right]$$

is used. The pressure, energy density, and entropy, are determined as polynomials in f with a set of precomputed coefficients as done in [Johns96](#).

If ψ is too small (less than about -200), the above procedure fails. To handle this, this class uses the classical result if $\psi < \text{min_psi}$, where [min_psi](#) defaults to -200.

When the density and temperature is given instead ([calc_density\(\)](#) and [pair_density\(\)](#)), then there are two ways to proceed.

- Use the density to solve for f .
- Use the density to solve for the chemical potential.

Because the density is a complicated polynomial in f , the former procedure does not work very well even though it might be less time consuming. In this class, the density is solved for the effective chemical potential instead. The initial guess is just taken from the present value of [part::nu](#).

Note

It is important to note that the coefficients are static and apply to all objects of type [eff_fermion](#).

The coefficients are stored in static data, and there is a small possibility of a memory leak in a multithreaded program if more than one instance of [eff_fermion](#) tries to initialize these coefficients in the constructor. This can easily be avoided by ensuring that the static coefficients are initialized by a single thread beforehand.

Todo There's still `def_err_hnd.set_mode(0)` in the testing code, probably because the solver has a hard time for extreme values.

Idea for Future Use bracketing to speed up one-dimensional root finding.

Definition at line 102 of file `eff_fermion.h`.

Data Structures

- class [density_fun](#)
Define the function which solves for the chemical potential given the density [protected subclass of [eff_fermion](#)].
- class [pair_density_fun](#)
Define the function which solves for the chemical potential given the density of particles and antiparticles [protected subclass of [eff_fermion](#)].

Public Member Functions

- [eff_fermion](#) ()
Create a fermion with mass `mass` and degeneracy `doF`.
- virtual void [calc_mu](#) ([fermion](#) &`f`, double `temper`)
Calculate thermodynamic properties as function of chemical potential.
- virtual void [calc_density](#) ([fermion](#) &`f`, double `temper`)
Calculate thermodynamic properties as function of density.
- virtual void [pair_mu](#) ([fermion](#) &`f`, double `temper`)
Calculate thermodynamic properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) ([fermion](#) &`f`, double `temper`)
Calculate thermodynamic properties with antiparticles as function of density.
- int [set_psi_root](#) (`root`< **funct** > &`rp`)
Set the solver for use in calculating ψ .
- int [set_density_root](#) (`root`< **funct** > &`rp`)
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * [type](#) ()
Return string denoting type ("eff_fermion")

Data Fields

- double [tlimit](#)
If the temperature is less than `tlimit` then the zero-temperature functions are used (default 0).
- **cern_mroot_root**< **funct** > [def_psi_root](#)
The default solver for ψ .
- **cern_mroot_root**< **funct** > [def_density_root](#)
The default solver for [calc_density\(\)](#) and [pair_density\(\)](#)
- double [min_psi](#)
The minimum value of ψ (default -200)

Protected Member Functions

- double [solve_fun](#) (double `x`, double &`psi`)
The function which solves for f from ψ .

Protected Attributes

- **umatrix** [Pmnf](#)
The matrix of coefficients.
- double [parma](#)
The parameter a .
- int [sizem](#)
The array row size.
- int [sizen](#)
The array column size.
- **root**< **funct** > * [psi_root](#)
The solver for ψ .
- **root**< **funct** > * [density_root](#)
The other solver for [calc_density\(\)](#)

Coefficients for finite-temperature approximation

- static const int `cf_fermitat3` = 1
A set of coefficients from Jim Lattimer.
- static const int `cf_fermijel2` = 2
The smaller set of coefficients from Johns96.
- static const int `cf_fermijel3` = 3
The larger set of coefficients from Johns96.
- static const int `cf_fermijel3cons` = 4
The set of coefficients from Johns96 which retains better thermodynamic consistency.
- void `load_coefficients` (int ctype)
Load coefficients.

8.9.2 Member Function Documentation

8.9.2.1 void eff_fermion::load_coefficients (int ctype)

The argument `ctype` Should be one of the constants below.

8.9.2.2 virtual void eff_fermion::calc.mu (fermion & f, double temper) [virtual]

If the quantity $(\mu - m)/T$ (or $(v - m^*)/T$ in the case of interacting particles) is less than -200, then this quietly sets the density, the scalar density, the energy density, the pressure and the entropy to zero and exits.

Todo Should see if the function actually works if $(\mu - m)/T = -199$.

Implements [fermion_eval_thermo](#).

8.9.2.3 virtual void eff_fermion::calc.density (fermion & f, double temper) [virtual]

Warning

This function needs a guess for the chemical potential, and will fail if that guess is not sufficiently accurate.

Implements [fermion_eval_thermo](#).

8.9.2.4 virtual void eff_fermion::pair_mu (fermion & f, double temper) [virtual]

Warning

This function needs a guess for the chemical potential, and will fail if that guess is not sufficiently accurate.

Implements [fermion_eval_thermo](#).

The documentation for this class was generated from the following file:

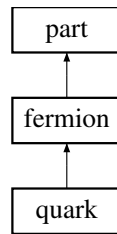
- `eff_fermion.h`

8.10 fermion Class Reference

Fermion class.

```
#include <fermion.h>
```

Inheritance diagram for fermion:



8.10.1 Detailed Description

This class adds two member data variables, `kf` and `del`, for the Fermi momentum and the gap, respectively.

Definition at line 49 of file `fermion.h`.

Public Member Functions

- `fermion` (double mass=0, double dof=0)
Create a fermion with mass `mass` and degeneracy `dof`.
- virtual const char * `type` ()
Return string denoting type ("fermion")

Data Fields

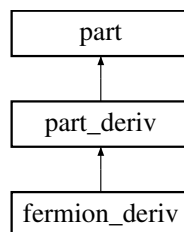
- double `kf`
Fermi momentum.
- double `del`
Gap.

The documentation for this class was generated from the following file:

- `fermion.h`

8.11 fermion_deriv Class Reference

Inheritance diagram for `fermion_deriv`:



8.11.1 Detailed Description

Definition at line 279 of file `deriv_part.h`.

Public Member Functions

- `fermion_deriv` (double mass=0.0, double dof=0.0)

Data Fields

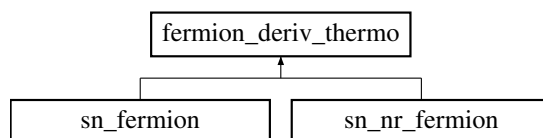
- double [kf](#)
Fermi momentum.

The documentation for this class was generated from the following file:

- [deriv_part.h](#)

8.12 fermion_deriv_thermo Class Reference

Inheritance diagram for fermion_deriv_thermo:



8.12.1 Detailed Description

Definition at line 291 of file [deriv_part.h](#).

Public Member Functions

- virtual void [calc_mu](#) ([fermion_deriv](#) &f, double temper)=0
Calculate properties as function of chemical potential.
- virtual void [calc_density](#) ([fermion_deriv](#) &f, double temper)=0
Calculate properties as function of density.
- virtual void [pair_mu](#) ([fermion_deriv](#) &f, double temper)=0
Calculate properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) ([fermion_deriv](#) &f, double temper)=0
Calculate properties with antiparticles as function of density.
- virtual void [nu_from_n](#) ([fermion_deriv](#) &f, double temper)=0
Calculate effective chemical potential from density.

The documentation for this class was generated from the following file:

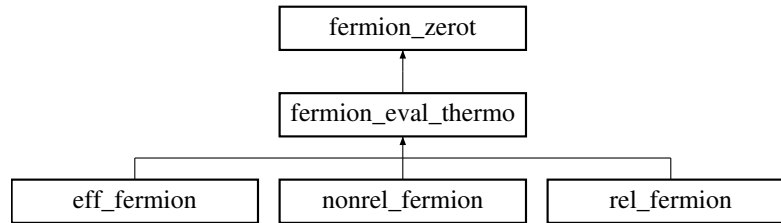
- [deriv_part.h](#)

8.13 fermion_eval_thermo Class Reference

Fermion with finite-temperature thermodynamics [abstract base].

```
#include <fermion.h>
```

Inheritance diagram for fermion_eval_thermo:



8.13.1 Detailed Description

This is an abstract base for the computation of finite-temperature fermionic statistics. Different children (e.g. [eff_fermion](#) and [rel_fermion](#)) use different techniques to computing the momentum integrations.

Because massless fermions at finite temperature are much simpler, there are separate member functions included in this class to handle them. The functions [massless_calc_density\(\)](#) and [massless_calc_mu\(\)](#) compute the thermodynamics of massless fermions at finite temperature given the density or the chemical potentials. The functions [massless_pair_density\(\)](#) and [massless_pair_mu\(\)](#) perform the same task, but automatically include antiparticles.

The function [massless_calc_density\(\)](#) uses a **root** object to solve for the chemical potential as a function of the density. The default is an object of type **cern_mroot_root**. The function [massless_pair_density\(\)](#) does not need to use the **root** object because of the simplification afforded by the inclusion of antiparticles.

Idea for Future Create a Chebyshev approximation for inverting the the Fermi functions for [massless_calc_density\(\)](#) functions?

This Mathematica notebook contains the derivations of related series expansions and some algebra for the [massless_pair\(\)](#) functions.

```
doc/o2scl/extras/fermion.nb
doc/o2scl/extras/fermion.pdf
```

Definition at line 181 of file fermion.h.

Public Member Functions

- virtual void [calc_mu](#) ([fermion](#) &f, double temper)=0
Calculate properties as function of chemical potential.
- virtual void [calc_density](#) ([fermion](#) &f, double temper)=0
Calculate properties as function of density.
- virtual void [pair_mu](#) ([fermion](#) &f, double temper)=0
Calculate properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) ([fermion](#) &f, double temper)=0
Calculate properties with antiparticles as function of density.
- void [set_massless_root](#) (**root**< **funct** > &rp)
Set the solver for use in [massless_calc_density\(\)](#)
- virtual const char * [type](#) ()
Return string denoting type ("fermion_eval_thermo")
- virtual double [calibrate](#) ([fermion](#) &f, int verbose=0, std::string fname="")
Test the thermodynamics of [calc_density\(\)](#) and [calc_mu\(\)](#)

Massless fermions

- virtual void [massless_calc_mu](#) ([fermion](#) &f, double temper)
Finite temperature massless fermions.
- virtual void [massless_calc_density](#) ([fermion](#) &f, double temper)
Finite temperature massless fermions.

- virtual void [massless_pair_mu](#) (fermion &f, double temper)
Finite temperature massless fermions and antifermions.
- virtual void [massless_pair_density](#) (fermion &f, double temper)
Finite temperature massless fermions and antifermions.

Data Fields

- **cern_mroot_root** < **funct** > [def_massless_root](#)
The default solver for [massless_calc_density\(\)](#)

8.13.2 Member Function Documentation

8.13.2.1 virtual void fermion_eval_thermo::massless_pair_density (fermion &f, double temper) [virtual]

In the cases $n^3 \gg T$ and $T \gg n^3$, expansions are used instead of the exact formulas to avoid loss of precision.

In particular, using the parameter

$$\alpha = \frac{g^2 \pi^2 T^6}{243 n^2}$$

and defining the expression

$$\text{cvt} = \alpha^{-1/6} \left(-1 + \sqrt{1 + \alpha} \right)^{1/3}$$

we can write the chemical potential as

$$\mu = \frac{\pi T}{\sqrt{3}} \left(\frac{1}{\text{cvt}} - \text{cvt} \right)$$

These expressions, however, do not work well when α is very large or very small, so series expansions are used whenever $\alpha > 10^4$ or $\alpha < 3 \times 10^{-4}$. For small α ,

$$\left(\frac{1}{\text{cvt}} - \text{cvt} \right) \approx \frac{2^{1/3}}{\alpha^{1/6}} - \frac{\alpha^{1/6}}{2^{1/3}} + \frac{\alpha^{5/6}}{6 \cdot 2^{2/3}} + \frac{\alpha^{7/6}}{12 \cdot 2^{1/3}} - \frac{\alpha^{11/6}}{18 \cdot 2^{2/3}} - \frac{5\alpha^{13/6}}{144 \cdot 2^{1/3}} + \frac{77\alpha^{17/6}}{2592 \cdot 2^{2/3}}$$

and for large α ,

$$\left(\frac{1}{\text{cvt}} - \text{cvt} \right) \approx \frac{2}{3} \sqrt{\frac{1}{\alpha}} - \frac{8}{81} \left(\frac{1}{\alpha} \right)^{3/2} + \frac{32}{729} \left(\frac{1}{\alpha} \right)^{5/2}$$

This approach works to within about 1 part in 10^{12} , and is tested in `fermion_ts.cpp`.

Idea for Future This could be improved by including more terms in the expansions.

8.13.2.2 virtual double fermion_eval_thermo::calibrate (fermion &f, int verbose = 0, std::string fname = " ") [virtual]

This compares the approximation to the exact results over a grid with $T = \{10^{-2}, 1, 10^2\}$, $\log_{10}(m/T) = \{-3, -2, -1, 0, 1, 2, 3\}$, and $\log_{10} \psi = \{-3, -2, -1, 0, 1\}$, where $\psi \equiv (\mu - m)/T$ using [calc_density\(\)](#) and [calc_mu\(\)](#), with both `inc_rest_mass` taking both values `true` and `false`.

The `verbose` parameter controls the amount of output, and `fname` is the filename for the file `fermion_cal.dat`.

Idea for Future Also calibrate massless fermions?

Idea for Future Convert into separate class?

8.13.3 Field Documentation

8.13.3.1 cern_mroot_root<funct> fermion_eval_thermo::def_massless_root

We default to a solver of type **cern_mroot_root** here since we don't have a bracket or a derivative.

Definition at line 284 of file fermion.h.

The documentation for this class was generated from the following file:

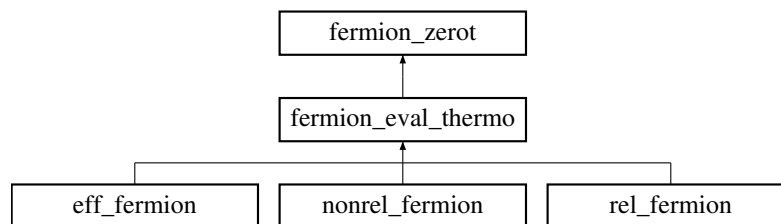
- fermion.h

8.14 fermion_zerot Class Reference

This is a base class for the computation of fermionic statistics at zero temperature. The more general case of finite temperature is taken care of by [fermion_eval_thermo](#) objects. The primary functions are [calc_mu_zerot\(\)](#) and [calc_density_zerot\(\)](#) which compute all the thermodynamic quantities as a function of the chemical potential, or the density, respectively.

```
#include <fermion.h>
```

Inheritance diagram for fermion_zerot:



8.14.1 Detailed Description

Idea for Future Use hypot() and other more accurate functions for the analytic expressions for the zero temperature integrals. [-Progress has been made, but there are probably other functions which may break down for small but finite masses and temperatures]

Definition at line 83 of file fermion.h.

Public Member Functions

Zero-temperature fermions

- void [kf_from_density](#) (fermion &f)
Calculate the Fermi momentum from the density.
- void [energy_density_zerot](#) (fermion &f)
Energy density at T=0 from *fermion::kf* and *part::ms*.
- void [pressure_zerot](#) (fermion &f)
Pressure at T=0 from *fermion::kf* and *part::ms*.
- virtual void [calc_mu_zerot](#) (fermion &f)
Zero temperature fermions from *part::nu* and *part::ms*.
- virtual void [calc_density_zerot](#) (fermion &f)
Zero temperature fermions from *part::n* and *part::ms*.

8.14.2 Member Function Documentation

8.14.2.1 void fermion_zerot::kf_from_density (fermion & f)

Uses the relation $k_F = (6\pi^2 n/g)^{1/3}$

8.14.2.2 void fermion_zerot::energy_density_zerot (fermion & f)

Calculates the integral

$$\varepsilon = \frac{g}{2\pi^2} \int_0^{k_F} k^2 \sqrt{k^2 + m^{*2}} dk$$

8.14.2.3 void fermion_zerot::pressure_zerot (fermion & f)

Calculates the integral

$$P = \frac{g}{6\pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m^{*2}}} dk$$

8.14.2.4 virtual void fermion_zerot::calc_mu_zerot (fermion & f) [virtual]

This function always returns `gsl_success`.

Reimplemented in [nonrel_fermion](#).

8.14.2.5 virtual void fermion_zerot::calc_density_zerot (fermion & f) [virtual]

This function always returns `gsl_success`.

Reimplemented in [nonrel_fermion](#).

The documentation for this class was generated from the following file:

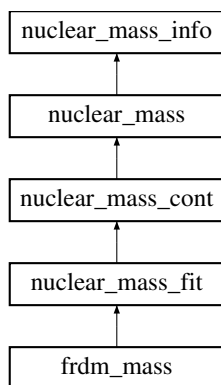
- `fermion.h`

8.15 frdm_mass Class Reference

FRDM semi-empirical mass formula (macroscopic part only with no deformation)

```
#include <frdm_mass.h>
```

Inheritance diagram for `frdm_mass`:



8.15.1 Detailed Description

The macroscopic part of the finite-range droplet model from [Moller95](#) .

Using the relations

$$\bar{\delta} = (n_n - n_p)/n$$

and

$$\bar{\epsilon} = -(n - n_0)/3/n_0$$

we get

$$n_n = \frac{1}{2}(1 + \bar{\delta})(1 - 3\bar{\epsilon})n_0$$

and

$$n_p = \frac{1}{2}(1 - \bar{\delta})(1 - 3\bar{\epsilon})n_0$$

Assuming that

$$\frac{4\pi}{3}R_n^3 n_n = N$$

and

$$\frac{4\pi}{3}R_p^3 n_p = Z$$

we get

$$R_n^3 = 3N/\alpha_n$$

$$R_p^3 = 3Z/\alpha_p$$

where α 's are

$$\alpha_n = 2\pi(1 + \bar{\delta})(1 - 3\bar{\epsilon})n_0$$

$$\alpha_p = 2\pi(1 - \bar{\delta})(1 - 3\bar{\epsilon})n_0$$

Note that the above relations are somehow self-consistent because they imply

$$R^3 n = R_n^3 n_n + R_p^3 n_p$$

Since we're using (is there a better way?)

$$R = r_0 A^{1/3}$$

with $r_0 = 1.16$ fm, then $n_0 = 0.152946 \text{ fm}^{-3}$.

Todo Fix pairing energy and double vs. int

Document drip_binding_energy(), etc.

Decide on number of fit parameters (10 or 12?) or let the user decide

Document the protected variables

Set the neutron and proton masses and hbarc to Moller et al.'s values

Idea for Future Add microscopic part.

Definition at line 103 of file frdm_mass.h.

Public Member Functions

- virtual double [mass_excess_d](#) (double Z, double N)
Given Z and N, return the mass excess in MeV.
- virtual int [fit_fun](#) (size_t nv, const **ovector_base** &x)
Fix parameters from an array for fitting.
- virtual int [guess_fun](#) (size_t nv, **ovector_base** &x)
Fill array with guess from present values for fitting.
- virtual double [drip_binding_energy_d](#) (double Z, double N, double npout, double nnout, double chi)
Return the binding energy in MeV.
- virtual double [drip_mass_excess_d](#) (double Z, double N, double np_out, double nn_out, double chi)
Given Z and N, return the mass excess in MeV.

Data Fields

- double [a1](#)
Volume-energy constant in MeV (default 16.247)
- double [J](#)
Symmetry-energy constant in MeV (default 32.73)
- double [K](#)
Nuclear compressibility constant in MeV (default 240)
- double [a2](#)
Surface-energy constant in MeV (default 22.92)
- double [Q](#)
Effective surface-stiffness constant in MeV (default 29.21)
- double [a3](#)
Curvature-energy constant in MeV (default 0)
- double [ca](#)
Charge-asymmetry constant in MeV (default 0.436)
- double [W](#)
Wigner constant in MeV (default 30)
- double [ael](#)
electronic-binding constant in MeV (default 1.433×10^{-5}).
- double [rp](#)
Proton root-mean-square radius in fm (default 0.80)
- double [r0](#)
Nuclear-radius constant in fm (default 1.16)
- double [MH](#)
Hydrogen atom mass excess, 7.289034 MeV.
- double [Mn](#)
Neutron mass excess, 8.071431 MeV.
- double [e2](#)
Electronic charge squared, 1.4399764 MeV fm.
- double [a](#)
Range of Yukawa-plus-exponential potential, 0.68 fm.
- double [aden](#)
Range of Yukawa function used to generate nuclear charge distribution, 0.70 fm.
- double [rmac](#)
Average pairing-gap constant, 4.80 MeV.
- double [h](#)
Neutron-proton interaction constant, 6.6 MeV.
- double [L](#)
Density-symmetry constant, 0 MeV.
- double [C](#)
Pre-exponential compressibility-term constant, 60 MeV.
- double [gamma](#)
Exponential compressibility-term range constant, 0.831.
- double [amu](#)
Atomic mass unit, 931.5014 MeV.
- double [nn](#)
Internal average neutron density.
- double [np](#)
Internal average proton density.
- double [Rn](#)
Neutron radius.
- double [Rp](#)
Proton radius.

Protected Attributes

- double [Deltap](#)
Proton pairing coefficient.
- double [Deltan](#)

- *Neutron pairing coefficient.*
- double [deltanp](#)
- *Isovector pairing coefficient.*
- double [deltabar](#)
- *Average bulk nuclear asymmetry.*
- double [epsbar](#)
- *Average relative deviation of bulk density.*
- double [Bs](#)
- *Desc.*
- double [Bk](#)
- *Desc.*
- double [Br](#)
- *Desc.*
- double [Bw](#)
- *Desc.*
- double [Bv](#)
- *Desc.*
- double [c1](#)
- *Coulomb energy coefficient.*
- double [c2](#)
- *Volume redistribution energy coefficient.*
- double [c4](#)
- *Coulomb exchange correction coefficient.*
- double [c5](#)
- *Surface redistribution energy coefficient.*
- double [f0](#)
- *Coefficient for the proton form-factor correction to the Coulomb energy.*
- double [a0](#)
- *Desc.*
- double [B1](#)
- *Desc.*
- double [B2](#)
- *Desc.*
- double [B3](#)
- *Desc.*
- double [B4](#)
- *Desc.*

The documentation for this class was generated from the following file:

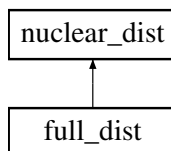
- frdm_mass.h

8.16 full_dist Class Reference

Full distribution including all nuclei from a discrete mass formula.

```
#include <nuclear_dist.h>
```

Inheritance diagram for full_dist:



8.16.1 Detailed Description

For example, to create a collection of all nuclei from the most recent (2003) Atomic Mass Evaluation, and then output all the nuclei in the collection

```
ame_mass ame;
full_dist fd(&ame);
for(nuclear_dist::iterator ndi=fd.begin();ndi!=fd.end();ndi++) {
    cout << ndi->Z << " " << ndi->A << " " << ndi->m << endl;
}
```

Definition at line 248 of file nuclear_dist.h.

Public Member Functions

- **full_dist** (**nuclear_mass** &nm, int maxA=400, bool include_neutron=false)
Create a distribution including all nuclei with atomic numbers less than maxA from the mass formula nm.
- int **set_dist** (**nuclear_mass** &nm, int maxA=400, bool include_neutron=false)
Set the distribution to all nuclei with atomic numbers less than maxA from the mass formula nm.
- virtual **iterator begin** ()
The beginning of the distribution.
- virtual **iterator end** ()
The end of the distribution.
- virtual **size_t size** ()
The number of nuclei in the distribution.

8.16.2 Member Function Documentation

8.16.2.1 int full_dist::set_dist (nuclear_mass & nm, int maxA = 400, bool include_neutron = false)

The information for the previous distribution is cleared before a new distribution is set.

The documentation for this class was generated from the following file:

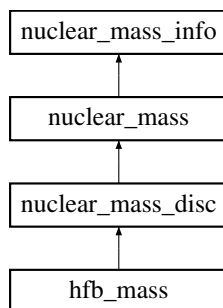
- nuclear_dist.h

8.17 hfb_mass Class Reference

HFB Mass formula.

```
#include <nuclear_mass.h>
```

Inheritance diagram for hfb_mass:



8.17.1 Detailed Description

Definition at line 1006 of file nuclear_mass.h.

Public Member Functions

- [hfb_mass](#) ()
Create a new mass formula object.
- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double [mass_excess](#) (int Z, int N)
Given Z and N, return the mass excess in MeV.
- [hfb_mass_entry](#) [get_ZN](#) (int l_Z, int l_N)
Get the entry for the specified proton and neutron number.
- bool [is_loaded](#) ()
Verify that the constructor properly loaded the table.
- double [blank](#) ()
The value which corresponds to a blank entry.
- virtual const char * [type](#) ()
Return the type, "hfb_mass".
- int [set_data](#) (int n_mass, [hfb_mass_entry](#) *m, std::string ref)
Set data.
- int [get_nentries](#) ()
Return number of entries.

Protected Attributes

- int [n](#)
The number of entries (about 3000).
- std::string [reference](#)
The reference for the original data.
- [hfb_mass_entry](#) * [mass](#)
The array containing the mass data of length ame::n.
- int [last](#)
The last table index for caching.

8.17.2 Member Function Documentation

8.17.2.1 hfb_mass_entry hfb_mass::get_ZN (int l_Z, int l_N)

This method searches the table using a cached binary search algorithm. It is assumed that the table is sorted first by proton number and then by neutron number.

8.17.2.2 int hfb_mass::set_data (int n_mass, hfb_mass_entry * m, std::string ref)

This function is used by the HDF I/O routines.

The documentation for this class was generated from the following file:

- nuclear_mass.h

8.18 hfb_mass_entry Struct Reference

Mass formula entry structure for HFB mass formula.

```
#include <nuclear_mass.h>
```

8.18.1 Detailed Description

Definition at line 907 of file nuclear_mass.h.

Data Fields

- int [N](#)
Neutron number.
- int [Z](#)
Proton number.
- int [A](#)
Atomic number.
- double [bet2](#)
Beta 2 deformation.
- double [bet4](#)
Beta 4 deformation.
- double [Rch](#)
RMS charge radius.
- double [def_wig](#)
Deformation and Wigner energies.
- double [Sn](#)
Neutron separation energy.
- double [Sp](#)
Proton separation energy.
- double [Qbet](#)
Beta-decay energy.
- double [Mcal](#)
Calculated mass excess.
- double [Err](#)
Error between experimental and calculated mass excess.

The documentation for this struct was generated from the following file:

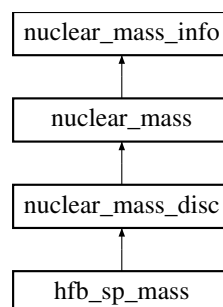
- nuclear_mass.h

8.19 hfb_sp_mass Class Reference

HFB Mass formula with spin and parity information.

```
#include <nuclear_mass.h>
```

Inheritance diagram for hfb_sp_mass:



8.19.1 Detailed Description

Definition at line 1072 of file nuclear_mass.h.

Public Member Functions

- [hfb_sp_mass](#) ()
Create a new mass formula object.
- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double [mass_excess](#) (int Z, int N)
Given Z and N, return the mass excess in MeV.
- [hfb_sp_mass_entry](#) [get_ZN](#) (int I_Z, int I_N)
Get the entry for the specified proton and neutron number.
- virtual const char * [type](#) ()
Return the type, "hfb_mass".
- int [set_data](#) (int n_mass, [hfb_sp_mass_entry](#) *m, std::string ref)
Set data.

Protected Attributes

- [hfb_sp_mass_entry](#) * [mass](#)
The array containing the mass data of length ame::n.
- int [n](#)
The number of entries (about 3000).
- std::string [reference](#)
The reference for the original data.
- int [last](#)
The last table index for caching.

8.19.2 Member Function Documentation

8.19.2.1 [hfb_sp_mass_entry](#) [hfb_sp_mass::get_ZN](#) (int I_Z, int I_N)

This method searches the table using a cached binary search algorithm. It is assumed that the table is sorted first by proton number and then by neutron number.

8.19.2.2 int [hfb_sp_mass::set_data](#) (int n_mass, [hfb_sp_mass_entry](#) * m, std::string ref)

This function is used by the HDF I/O routines.

The documentation for this class was generated from the following file:

- [nuclear_mass.h](#)

8.20 hfb_sp_mass_entry Struct Reference

Version of [hfb_mass_entry](#) with spin and parity.

```
#include <nuclear_mass.h>
```

8.20.1 Detailed Description

Note

This cannot be a child of [hfb_mass_entry](#) in order for the HDF I/O preprocessor macros, like HOFFSET, to work

Definition at line 952 of file [nuclear_mass.h](#).

Data Fields

- int [N](#)
Neutron number.
- int [Z](#)
Proton number.
- int [A](#)
Atomic number.
- double [bet2](#)
Beta 2 deformation.
- double [bet4](#)
Beta 4 deformation.
- double [Rch](#)
RMS charge radius.
- double [def_wig](#)
Deformation and Wigner energies.
- double [Sn](#)
Neutron separation energy.
- double [Sp](#)
Proton separation energy.
- double [Qbet](#)
Beta-decay energy.
- double [Mcal](#)
Calculated mass excess.
- double [Err](#)
Error between experimental and calculated mass excess.
- double [Jexp](#)
Experimental spin.
- double [Jth](#)
Theoretical spin.
- int [Pexp](#)
Experimental parity.
- int [Pth](#)
Theoretical parity.

The documentation for this struct was generated from the following file:

- [nuclear_mass.h](#)

8.21 nuclear_dist::iterator Class Reference

An iterator for the nuclear distribution.

```
#include <nuclear_dist.h>
```

8.21.1 Detailed Description

The standard usage of this iterator is something of the form:

```
mnmsk_mass mth;
simple_dist sd(5,6,10,12,&mth);
for(nuclear_dist::iterator ndi=sd.begin();ndi!=sd.end();ndi++) {
    // do something here for each nucleus
}
```

which would create a list consisting of three isotopes (A=10, 11, and 12) of boron and three isotopes carbon for a total of six nuclei.

Definition at line 69 of file [nuclear_dist.h](#).

Public Member Functions

- `iterator (nuclear_dist *ndpp, nucleus *npp)`
Create an iterator from the given distribution using the nucleus specified in npp.
- `iterator operator++ ()`
Proceed to the next nucleus.
- `iterator operator++ (int unused)`
Proceed to the next nucleus.
- `nucleus * operator-> () const`
Pointing at operator.
- `nucleus & operator* () const`
Dereference the iterator.

Protected Attributes

- `nucleus * np`
A pointer to the current nucleus.
- `nuclear_dist * ndp`
A pointer to the distribution.

Friends

- `bool operator== (const nuclear_dist::iterator &i1, const nuclear_dist::iterator &i2)`
Give access to the == operator.
- `bool operator!= (const nuclear_dist::iterator &i1, const nuclear_dist::iterator &i2)`
Give access to the != operator.

The documentation for this class was generated from the following file:

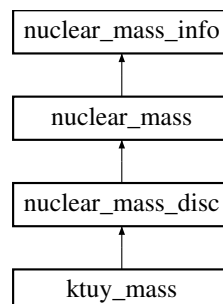
- `nuclear_dist.h`

8.22 ktuy_mass Class Reference

KTUY Mass formula.

```
#include <nuclear_mass.h>
```

Inheritance diagram for ktuy_mass:



8.22.1 Detailed Description

Definition at line 1208 of file nuclear_mass.h.

Public Member Functions

- `ktuy_mass` (std::string model="05", bool external=false)
Create a new mass formula object using the specified model number.
- virtual bool `is_included` (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double `mass_excess` (int Z, int N)
Given Z and N, return the mass excess in MeV.
- `ktuy_mass_entry` `get_ZN` (int l_Z, int l_N)
Get the entry for the specified proton and neutron number.
- bool `is_loaded` ()
Verify that the constructor properly loaded the table.
- virtual const char * `type` ()
Return the type, "ktuy_mass".
- int `get_nentries` ()
Return number of entries.

Protected Attributes

- int `n`
The number of entries (about 3000).
- std::string `reference`
The reference for the original data.
- `ktuy_mass_entry` * `mass`
The array containing the mass data of length ame::n.
- int `last`
The last table index for caching.

8.22.2 Member Function Documentation

8.22.2.1 ktuy_mass_entry ktuy_mass::get_ZN(int l_Z, int l_N)

This method searches the table using a cached binary search algorithm. It is assumed that the table is sorted first by proton number and then by neutron number.

The documentation for this class was generated from the following file:

- nuclear_mass.h

8.23 ktuy_mass_entry Struct Reference

Mass formula entry structure for KTUY mass formula.

```
#include <nuclear_mass.h>
```

8.23.1 Detailed Description

Definition at line 1178 of file nuclear_mass.h.

Data Fields

- int `N`
Neutron number.
- int `Z`
Proton number.

- int [A](#)
Atomic number.
- double [Mcal](#)
Calculated mass excess.
- double [Esh](#)
Shell energy.
- double [alpha2](#)
Alpha 2 deformation.
- double [alpha4](#)
Alpha 4 deformation.
- double [alpha6](#)
Alpha 6 deformation.

The documentation for this struct was generated from the following file:

- `nuclear_mass.h`

8.24 mass_fit Class Reference

Fit a nuclear mass formula.

```
#include <mass_fit.h>
```

8.24.1 Detailed Description

There is an example of the usage of this class given in [Nuclear mass fit example](#).

Idea for Future Convert to a real fit with errors and covariance, etc.

Definition at line 44 of file `mass_fit.h`.

Public Member Functions

- virtual int [fit](#) ([nuclear_mass_fit](#) &n, double &res)
Fit the nuclear mass formula.
- virtual int [eval](#) ([nuclear_mass](#) &n, double &res)
Evaluate quality without fitting.
- int [set_mmin](#) ([multi_min](#)< [multi_func](#)<> > &umm)
Change the minimizer for use in the fit.
- int [set_dist](#) ([nuclear_dist](#) &uexp)
Set the distribution of nuclei to fit.
- int [set_exp_mass](#) ([nuclear_mass](#) &nm)
Set the experimental nuclear mass formula.

Data Fields

- bool [even_even](#)
If true, then only fit doubly-even nuclei (default false)
- int [minZ](#)
Minimum proton number to fit (default 8)
- int [minN](#)
Minimum neutron number to fit (default 8)
- [gsl_mmin_simp2](#)< [multi_func](#)<> > [def_mmin](#)
The default minimizer.
- [full_dist](#) [def_dist](#)
The default distribution of nuclei to fit (defaults to all nuclei in [def_exp_mass](#))
- [ame_mass](#) [def_exp_mass](#)
The default experimental nuclear mass object for [def_dist](#).

8.24.2 Field Documentation

8.24.2.1 gsl_mmin_simp2<multi_funct<>> mass_fit::def_mmin

The value of `def_mmin::ntrial` is automatically multiplied by 10 in the constructor because the minimization frequently requires more trials than the default.

Definition at line 73 of file `mass_fit.h`.

The documentation for this class was generated from the following file:

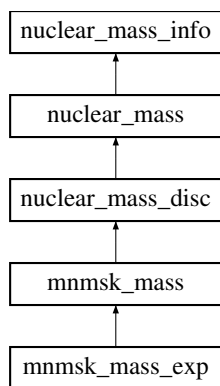
- `mass_fit.h`

8.25 mnmsk_mass Class Reference

Mass formula from Moller, Nix, Myers, Swiatecki and Kratz.

```
#include <nuclear_mass.h>
```

Inheritance diagram for `mnmsk_mass`:



8.25.1 Detailed Description

This is based on the tables given in [Moller95](#) and [Moller97](#).

Note

This class requires data stored in an HDF file and thus requires HDF support for normal usage.

The data containing an object of type `moller_mass_entry` for 8979 nuclei is automatically loaded by the constructor. If the file (`nucmass/mnmsk.o2`) is not found, then `is_loaded()` will return `false` and all calls to `get_ZN()` will return an object with `N=Z=0`.

There are several entries in the original table which are blank because they are in some way not known, measured, or computable. To distinguish these values from zero, blank entries have been replaced by the number `1.0e99`. For convenience, this value is returned by `blank()`.

Definition at line 816 of file `nuclear_mass.h`.

Public Member Functions

- virtual bool `is_included` (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double `mass_excess` (int Z, int N)

Given Z and N , return the mass excess in MeV.

- `mnmsk_mass_entry get_ZN` (int l_Z , int l_N)
Get the entry for the specified proton and neutron number.
- double `blank` ()
The value which corresponds to a blank entry.
- double `neither` ()
Neither beta+ or beta- is possible.
- double `beta_stable` ()
The value which corresponds to a blank entry.
- double `beta_plus_and_minus` ()
Both beta+ and beta- are possible.
- double `greater_100` ()
The value is greater than 100.
- double `very_large` ()
The value is greater than 10^{20} .
- virtual const char * `type` ()
Return the type, "mnmsk_mass".
- int `set_data` (int n_{mass} , `mnmsk_mass_entry` * m , std::string ref)
Set data.

Protected Attributes

- int `n`
The number of entries (about 3000).
- std::string `reference`
The reference for the original data.
- `mnmsk_mass_entry` * `mass`
The array containing the mass data of length $ame::n$.
- int `last`
The last table index for caching.

8.25.2 Member Function Documentation

8.25.2.1 mnmsk_mass_entry mnmsk_mass::get_ZN (int l_Z , int l_N)

This method searches the table using a cached binary search algorithm. It is assumed that the table is sorted first by proton number and then by neutron number.

8.25.2.2 int mnmsk_mass::set_data (int n_{mass} , `mnmsk_mass_entry` * m , std::string ref)

This function is used by the HDF I/O routines.

The documentation for this class was generated from the following file:

- nuclear_mass.h

8.26 mnmsk_mass_entry Struct Reference

Mass formula entry structure for Moller, et al.

```
#include <nuclear_mass.h>
```

8.26.1 Detailed Description

Definition at line 687 of file nuclear_mass.h.

Data Fields

- int [N](#)
Neutron number.
- int [Z](#)
Proton number.
- int [A](#)
Atomic number.
- double [Emic](#)
The ground-state microscopic energy.
- double [Mth](#)
The theoretical mass excess (in MeV)
- double [Mexp](#)
The experimental mass excess (in MeV)
- double [sigmaexp](#)
Experimental mass excess error.
- double [EmicFL](#)
The ground-state microscopic energy in the FRLDM.
- double [MthFL](#)
The theoretical mass excess in the FRLDM.
- char [spinp](#) [6]
Spin and parity of odd proton.
- char [spinn](#) [6]
Spin and parity of odd neutron.
- double [gapp](#)
Lipkin-Nogami proton gap.
- double [gapn](#)
Lipkin-Nogami neutron gap.
- double [be](#)
Total binding energy.
- double [S1n](#)
One neutron separation energy.
- double [S2n](#)
Two neutron separation energy.
- double [PA](#)
Percentage of daughters generated in beta decay after beta-delayed neutron emission.
- double [PAm1](#)
Desc.
- double [PAm2](#)
Desc.
- double [Qbeta](#)
Energy released in beta-decay.
- double [Tbeta](#)
Half-life w.r.t. GT beta-decay.
- double [S1p](#)
One proton separation energy.
- double [S2p](#)
Two proton separation energy.
- double [Qalpha](#)
Energy released in alpha-decay.
- double [Talpha](#)
Half-life w.r.t. alpha-decay.

Ground state deformations (perturbed-spheroid parameterization)

- double [eps2](#)
Quadrupole.
- double [eps3](#)
Octupole.
- double [eps4](#)
Hexadecapole.

- double [eps6](#)
Hexacontatetrapole.
- double [eps6sym](#)
Hexacontatetrapole without mass asymmetry.

Ground state deformations in the spherical-harmonics expansion

- double [beta2](#)
Quadrupole.
- double [beta3](#)
Octupole.
- double [beta4](#)
Hexadecapole.
- double [beta6](#)
Hexacontatetrapole.

The documentation for this struct was generated from the following file:

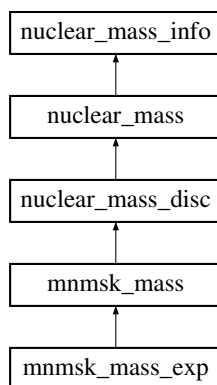
- `nuclear_mass.h`

8.27 mnmsk_mass_exp Class Reference

The experimental values from Moller, Nix, Myers and Swiatecki.

```
#include <nuclear_mass.h>
```

Inheritance diagram for `mnmsk_mass_exp`:



8.27.1 Detailed Description

Note

This class requires data stored in an HDF file and thus requires HDF support for normal usage.

Definition at line 891 of file `nuclear_mass.h`.

Public Member Functions

- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual double [mass_excess](#) (int Z, int N)
Given Z and N, return the mass excess in MeV.

The documentation for this class was generated from the following file:

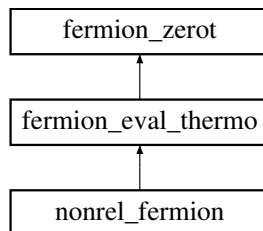
- `nuclear_mass.h`

8.28 nonrel_fermion Class Reference

Nonrelativistic fermion class.

```
#include <nonrel_fermion.h>
```

Inheritance diagram for nonrel_fermion:



8.28.1 Detailed Description

The rest mass energy density is given by $n \cdot m$ not $n \cdot m_s$. Note that the effective mass here is the Landau mass, not the Dirac mass.

Pressure is computed with

$$P = 2\varepsilon/3$$

and entropy density with

$$s = \frac{5\varepsilon}{3T} - \frac{n\mu}{T}$$

These relations can be verified with an integration by parts. See, e.g. [Callen](#) pg. 403 or [Landau](#) pg. 164.

The functions `fermion::pair_density()` and `pair_mu()` have not been implemented.

Todo Check behaviour of `calc_density()` at zero density, and compare with that from `eff_fermion`, `rel_fermion`, and `classical`.

Todo Implement `pair_density()` and `pair_mu()`.

Todo Make sure to test with `non-interacting` equal to true or false, and document whether or not it works with both `inc_rest_mass` equal to true or false

Idea for Future This could be improved by performing a Chebyshev approximation (for example) to invert the density integral so that we don't need to use a solver.

Definition at line 75 of file `nonrel_fermion.h`.

Public Member Functions

- `nonrel_fermion()`
Create a nonrelativistic fermion with mass 'm' and degeneracy 'g'.
- virtual void `calc_mu_zerot(fermion &f)`
Zero temperature fermions.
- virtual void `calc_density_zerot(fermion &f)`
Zero temperature fermions.
- virtual void `calc_mu(fermion &f, double temper)`
Calculate properties as function of chemical potential.
- virtual void `calc_density(fermion &f, double temper)`
Calculate properties as function of density.
- virtual void `pair_mu(fermion &f, double temper)`

- Calculate properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) ([fermion](#) &f, double temper)
Calculate properties with antiparticles as function of density.
- virtual void [nu_from_n](#) ([fermion](#) &f, double temper)
Calculate effective chemical potential from density.
- int [set_density_root](#) ([root](#)< [funct](#) > &rp)
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * [type](#) ()
Return string denoting type ("nonrel_fermion")

Data Fields

- [cern_mroot_root](#)< [funct](#) > [def_density_root](#)
The default solver for [calc_density\(\)](#).

8.28.2 Member Function Documentation

8.28.2.1 virtual void nonrel_fermion::calc_density ([fermion](#) &f, double temper) [virtual]

If the density is zero, this function just sets [part::mu](#), [part::nu](#), [part::ed](#), [part::pr](#), and [part::en](#) to zero and returns without calling the error handler (even though at zero density and finite temperature, the chemical potentials formally are equal to $-\infty$).

Implements [fermion_eval_thermo](#).

The documentation for this class was generated from the following file:

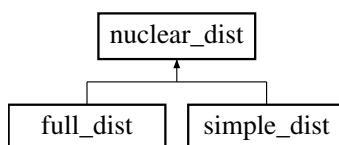
- nonrel_fermion.h

8.29 nuclear_dist Class Reference

A distribution of nuclei [abstract base].

```
#include <nuclear_dist.h>
```

Inheritance diagram for nuclear_dist:



8.29.1 Detailed Description

The virtual base class for a collection of objects of type [nucleus](#) . See [simple_dist](#) and [full_dist](#) for implementations of this base class.

Definition at line 40 of file nuclear_dist.h.

Data Structures

- class [iterator](#)
An iterator for the nuclear distribution.

Public Member Functions

- virtual [iterator begin](#) ()=0
The beginning of the distribution.
- virtual [iterator end](#) ()=0
The end of the distribution.
- virtual [size_t size](#) ()=0
The number of nuclei in the distribution.

The documentation for this class was generated from the following file:

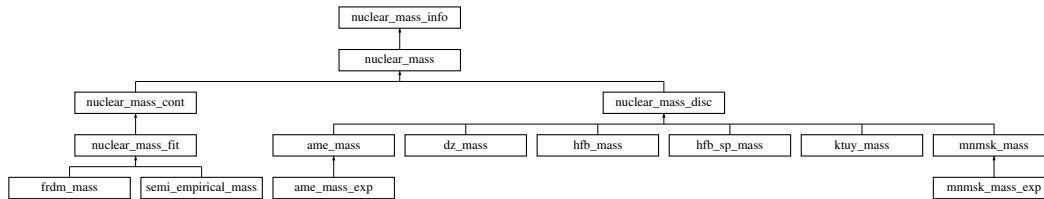
- `nuclear_dist.h`

8.30 nuclear_mass Class Reference

Nuclear mass formula base [abstract base].

```
#include <nuclear_mass.h>
```

Inheritance diagram for `nuclear_mass`:



8.30.1 Detailed Description

This base class provides some default functionality for the nuclear mass formulas. For typical usage, use [ame_mass](#), [mnmsk_mass](#), [mnmsk_mass_exp](#), or [semi_empirical_mass](#).

Elements 113, 115, 117 and 118 are named "Uut", "Uup", "Uus", and "Uuo", respectively.

Binding energies are determined from mass excesses by

$$\text{binding energy} = Au - Z(m_p + m_e) - Nm_n + \text{mass excess}$$

where u , m_n , m_p and m_e are all in `o2scl_fm`. "Total masses" are the mass of the nuclide (without the electron mass contribution)

$$\text{total mass} = \text{mass excess} + Au - Zm_e$$

Note that `O2scl` generally ignores contributions from electron binding energies.

Some mass formulas are undefined for sufficiently exotic nuclei. You can use the function [is_included\(\)](#) to find if a particular nucleus is included or not in a particular mass formula.

Generally, descendants of this class only need to provide an implementation of [mass_excess\(\)](#) and [mass_excess_d\(\)](#) and possibly a new version of [is_included\(\)](#).

Some common reaction Q-values and separation energies:

$$Q(\beta^-) = M(A, Z) - M(A, Z + 1): \text{Beta-decay energy}$$

$$Q(2\beta^-) = M(A, Z) - M(A, Z + 2): \text{Double beta-decay energy}$$

$$Q(4\beta^-) = M(A, Z) - M(A, Z + 4): \text{Four beta-decay energy}$$

$$Q(\alpha) = M(A, Z) - M(A - 4, Z - 2) - M(\text{He}^4): \text{Alpha-decay energy}$$

$Q(\beta - n) = M(A, Z) - M(A - 1, Z + 1) - M(n)$: Beta-delayed neutron emission decay energy

$Q(d, \alpha) = M(A, Z) - M(A - 2, Z - 1) - M(\text{He}^4) - M(\text{H}^2)$: (d, α) reaction energy

$Q(\text{EC}) = M(A, Z) - M(A, Z - 1)$: Electron capture decay energy

$Q(\text{ECp}) = M(A, Z) - M(A - 1, Z - 2)$: Electron capture with delayed proton emission decay energy

$Q(n, \alpha) = M(A, Z) - M(A - 3, Z - 2) - M(\text{He}^4) + M(n)$: (n, α) reaction energy

$Q(p, \alpha) = M(A, Z) - M(A - 3, Z - 1) - M(\text{He}^4) + M(p)$: (p, α) reaction energy

$S(n) = -M(A, Z) + M(A - 1, Z) + M(n)$: Neutron separation energy

$S(p) = -M(A, Z) + M(A - 1, Z - 1) + H^1$: Proton separation energy

$S(2n) = -M(A, Z) + M(A - 2, Z) + 2M(n)$: Two neutron separation energy

$S(2p) = -M(A, Z) + M(A - 2, Z - 2) + 2M(H^1)$: Two proton separation energy

Idea for Future Find a way to include the electron binding energy contribution, possibly with the help of a theoretical model.

Definition at line 192 of file nuclear_mass.h.

Public Member Functions

- virtual const char * [type](#) ()
Return the type, "nuclear_mass".
- virtual bool [is_included](#) (int Z, int N)
Return false if the mass formula does not include specified nucleus.
- virtual int [get_nucleus](#) (int Z, int N, [nucleus](#) &n)
Fill n with the information from nucleus with the given neutron and proton number.
- virtual double [mass_excess](#) (int Z, int N)=0
Given Z and N, return the mass excess in MeV.
- virtual double [mass_excess_d](#) (double Z, double N)=0
Given Z and N, return the mass excess in MeV.
- virtual double [electron_binding](#) (double Z)
Return the approximate electron binding energy in MeV.
- virtual double [binding_energy](#) (int Z, int N)
Return the binding energy in MeV.
- virtual double [binding_energy_d](#) (double Z, double N)
Return the binding energy in MeV.
- virtual double [total_mass](#) (int Z, int N)
Return the total mass of the nucleus (without the electrons) in MeV.
- virtual double [total_mass_d](#) (double Z, double N)
Return the total mass of the nucleus (without the electrons) in MeV.
- virtual double [atomic_mass](#) (int Z, int N)
Return the atomic mass of the nucleus in MeV (includes electrons and their binding energy)
- virtual double [atomic_mass_d](#) (double Z, double N)
Return the atomic mass of the nucleus in MeV (includes electrons and their binding energy)

Data Fields

- double [m_neut](#)
Neutron mass in MeV (defaults to `o2scl_fm::mass_neutron` times `o2scl_const::hc_mev_fm`)
- double [m_prot](#)
Proton mass in MeV (defaults to `o2scl_fm::mass_proton` times `o2scl_const::hc_mev_fm`)
- double [m_elec](#)
Electron mass in MeV (defaults to `o2scl_fm::mass_elec` times `o2scl_const::hc_mev_fm`)
- double [m_amu](#)
Atomic mass unit in MeV (defaults to `o2scl_fm::mass_amu` times `o2scl_const::hc_mev_fm`)

8.30.2 Member Function Documentation

8.30.2.1 virtual int nuclear_mass::get_nucleus (int Z, int N, nucleus & n) [virtual]

All masses are given in fm^{-1} . The total mass (withouth the electrons) is put in [part::m](#) and [part::ms](#), the binding energy is placed in [nucleus::be](#), the mass excess in [nucleus::mex](#) and the degeneracy ([part::g](#)) is arbitrarily set to 1 for even A nuclei and 2 for odd A nuclei.

8.30.2.2 virtual double nuclear_mass::binding_energy (int Z, int N) [inline, virtual]

The binding energy is defined to be negative for bound nuclei, thus the binding energy per baryon of Pb-208 is about $-8 \times 208 = -1664$ MeV.

Definition at line 257 of file nuclear_mass.h.

8.30.2.3 virtual double nuclear_mass::binding_energy_d (double Z, double N) [inline, virtual]

The binding energy is defined to be negative for bound nuclei, thus the binding energy per baryon of Pb-208 is about $-8 \times 208 = -1664$ MeV.

Definition at line 267 of file nuclear_mass.h.

The documentation for this class was generated from the following file:

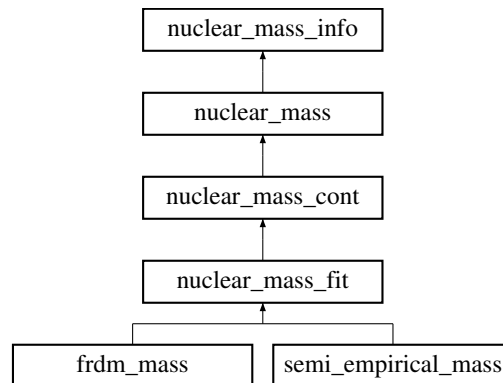
- nuclear_mass.h

8.31 nuclear_mass_cont Class Reference

Continuous nuclear mass formula [abstract base].

```
#include <nuclear_mass.h>
```

Inheritance diagram for nuclear_mass_cont:



8.31.1 Detailed Description

Generally, descendants of this class only need to provide an implementation of [mass_excess_d\(\)](#) and possibly a version of [nuclear_mass::is_included\(\)](#)

Definition at line 330 of file nuclear_mass.h.

Public Member Functions

- virtual double [mass_excess](#) (int Z, int N)

Given Z and N , return the mass excess in MeV.

- virtual double [mass_excess_d](#) (double Z , double N)=0

Given Z and N , return the mass excess in MeV.

The documentation for this class was generated from the following file:

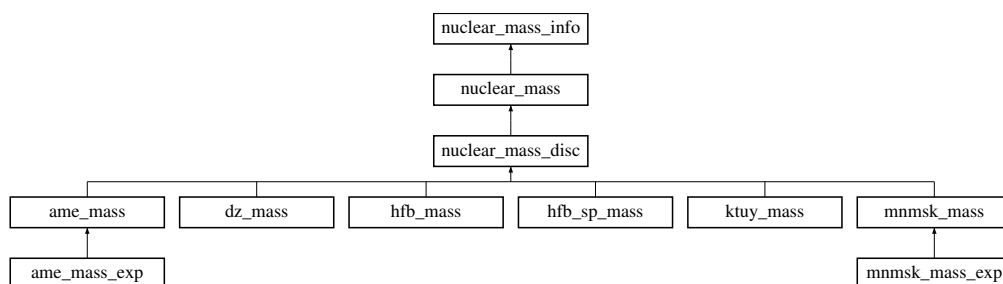
- nuclear_mass.h

8.32 nuclear_mass_disc Class Reference

Discrete nuclear mass formula [abstract base].

```
#include <nuclear_mass.h>
```

Inheritance diagram for nuclear_mass_disc:



8.32.1 Detailed Description

This uses simple linear interpolation to obtain masses of nuclei with non-integer Z and N which may be particularly sensitive to the form of the pairing.

Generally, descendants of this class only need to provide an implementation of [mass_excess\(\)](#) and possibly a version of [nuclear_mass::is_included\(\)](#)

Definition at line 312 of file nuclear_mass.h.

Public Member Functions

- virtual double [mass_excess](#) (int Z , int N)=0
Given Z and N , return the mass excess in MeV.
- virtual double [mass_excess_d](#) (double Z , double N)
Given Z and N , return the mass excess in MeV.

The documentation for this class was generated from the following file:

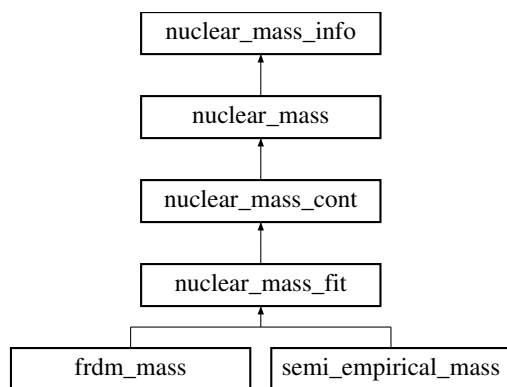
- nuclear_mass.h

8.33 nuclear_mass_fit Class Reference

Fittable mass formula.

```
#include <nuclear_mass.h>
```

Inheritance diagram for nuclear_mass_fit:



8.33.1 Detailed Description

Nuclear mass formulas which are descendants of this class can be fit to experiment using [mass_fit](#).

Within `O2scl_part`, this class has only two children, [frdm_mass](#) and [semi_empirical_mass](#). There is also a child `ldrop_mass` in `O2scl_eos`.

Idea for Future This shouldn't be a child of [nuclear_mass_cont](#)?

Definition at line 355 of file `nuclear_mass.h`.

Public Member Functions

- virtual const char * [type](#) ()
Return the type, "nuclear_mass_fit".
- virtual int [fit_fun](#) (size_t nv, const **ovector_base** &x)=0
Fix parameters from an array for fitting.
- virtual int [guess_fun](#) (size_t nv, **ovector_base** &x)=0
Fill array with guess from present values for fitting.

Data Fields

- size_t [nfit](#)
Number of fitting parameters.

The documentation for this class was generated from the following file:

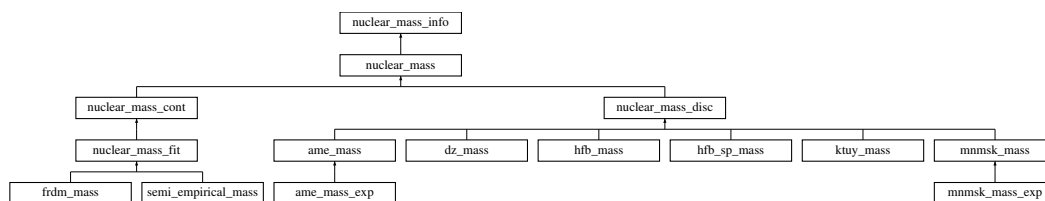
- `nuclear_mass.h`

8.34 nuclear_mass_info Class Reference

Nuclear mass info.

```
#include <nuclear_mass.h>
```

Inheritance diagram for `nuclear_mass_info`:



8.34.1 Detailed Description

Definition at line 38 of file nuclear_mass.h.

Public Member Functions

- int [parse_elstring](#) (std::string ela, int &Z, int &N, int &A)
Parse a string representing an element.
- int [eltoZ](#) (std::string el)
Return Z given the element name.
- std::string [Ztoel](#) (size_t Z)
Return the element name given Z.
- std::string [tostring](#) (size_t Z, size_t N)
Return a string of the form "Pb208" for a given Z and N.

Protected Types

- typedef std::map< std::string, int, **string_comp** >::iterator [table_it](#)
A convenient typedef for an iterator for element_table.

Protected Attributes

- std::map< std::string, int, **string_comp** > [element_table](#)
A map containing the proton numbers organized by element name.
- std::string [element_list](#) [[nelements](#)]
The list of elements organized by proton number.

Static Protected Attributes

- static const int [nelements](#) = 119
The number of elements (proton number)

8.34.2 Member Function Documentation

8.34.2.1 int nuclear_mass.info::parse_elstring (std::string ela, int & Z, int & N, int & A)

Accepts strings of one of the following forms:

- Pb208
- pb208
- Pb 208
- Pb-208

- `pb 208`
- `pb-208` or one of the special strings `n`, `p`, `d` or `t` for the neutron, proton, deuteron, and triton, respectively. This function also allows the value of `A` to precede the element symbol.

Note

At present, this allows nuclei which don't make sense because $A < Z$, such as Carbon-5.

Idea for Future Warn about malformed combinations like Carbon-5

Idea for Future Right now, `n4` is interpreted incorrectly as Nitrogen-4, rather than the tetraneutron.

8.34.2.2 `int nuclear_mass_info::eltoZ (std::string el)`

If the string parameter `el` is invalid, the error handler is called and the value -1 is returned.

8.34.2.3 `std::string nuclear_mass_info::Ztoel (size_t Z)`

Note

This function returns "n" indicating the neutron for $Z=0$, and if the argument Z is greater than 118, an empty string is returned after calling the error handler.

8.34.2.4 `std::string nuclear_mass_info::tostring (size_t Z, size_t N)`

Note that if Z is zero, then and 'n' is used to indicate the a nucleus composed entirely of neutrons and if the argument Z is greater than 118, an empty string is returned (independ.

The documentation for this class was generated from the following file:

- `nuclear_mass.h`

8.35 nuclear_reaction Class Reference

A simple nuclear reaction specification.

```
#include <reaction_lib.h>
```

8.35.1 Detailed Description

This class is very experimental.

Definition at line 44 of file `reaction_lib.h`.

Public Member Functions

- `std::string to_string ()`
Convert the reaction to a string for screen output.
- `int clear ()`
Clear the rate.
- `nuclear_reaction (const nuclear_reaction &nr)`
Copy constructor.
- `nuclear_reaction & operator= (const nuclear_reaction &nr)`
Copy constructor.
- `double rate (double T9)`
Compute the reaction rate from the temperature in units of $10^9 K$.

Data Fields

- size_t [chap](#)
Chapter.
- std::string [name](#) [6]
Names of the participating nuclei.
- std::string [ref](#)
Reference.
- char [type](#)
Type of rate (resonant/non-resonant/weak)
- char [rev](#)
Forward or reverse.
- double [Q](#)
Q value.
- double [a](#) [7]
Coefficients.
- size_t [Z](#) [6]
Proton number of participating nuclei.
- size_t [A](#) [6]
Mass number of participating nuclei.
- size_t [isomer](#) [6]
Isomer designation of participating nuclei.

The documentation for this class was generated from the following file:

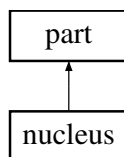
- [reaction_lib.h](#)

8.36 nucleus Class Reference

A simple nucleus class.

```
#include <nucleus.h>
```

Inheritance diagram for nucleus:



8.36.1 Detailed Description

The variable [part::m](#) is typically used for the mass of the nucleus with no electrons.

The binding energy of the nucleus ([be](#)) is typically defined as the mass of the nucleus (without the electrons) minus Z times the mass of the proton minus N times the mass of the neutron.

The mass excess ([be](#)) is defined as the mass of the nucleus including the electron contribution minus a times the mass of the atomic mass unit.

The variable [part::inc_rest_mass](#) is set to `false` by default, to insure that energies and chemical potentials do not include the rest mass. This is typically appropriate for nuclei.

Definition at line 50 of file [nucleus.h](#).

Data Fields

- int [Z](#)
Proton number.
- int [N](#)
Neutron number.
- int [A](#)
Atomic number.
- double [mex](#)
Mass excess in fm⁻¹.
- double [be](#)
Binding energy in fm⁻¹ (with a minus sign for bound nuclei)

The documentation for this class was generated from the following file:

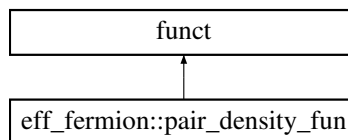
- nucleus.h

8.37 eff_fermion::pair_density_fun Class Reference

Define the function which solves for the chemical potential given the density of particles and antiparticles [protected subclass of [eff_fermion](#)].

```
#include <eff_fermion.h>
```

Inheritance diagram for eff_fermion::pair_density_fun:



8.37.1 Detailed Description

Definition at line 248 of file eff_fermion.h.

Public Member Functions

- **pair_density_fun** ([eff_fermion](#) &ef, [fermion](#) &f, double T)
- double [operator\(\)](#) (double x)
Fix density for [eff_fermion::pair_density\(\)](#)

Protected Attributes

- [eff_fermion](#) & **ef_**
- [fermion](#) & **f_**
- double **T_**

The documentation for this class was generated from the following file:

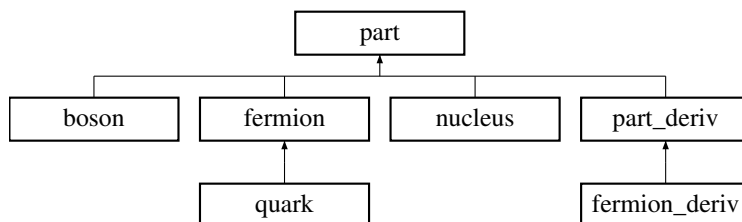
- eff_fermion.h

8.38 part Class Reference

Particle base class.

```
#include <part.h>
```

Inheritance diagram for part:



8.38.1 Detailed Description

Calculate the properties of particles from their chemical potential (`calc_mu()` and `pair_mu()`) or from the density (`calc_density()` and `pair_density()`).

When non-interacting is false, the thermodynamic integrals need both a value of "mu" and "nu". "nu" is an effective chemical potential which appears in the argument of the exponential of the Fermi-function.

Keep in mind, that the pair functions use `anti()`, which assumes that $\nu \rightarrow -\nu$ and $\mu \rightarrow -\mu$ for the anti-particles, which might not be true for interacting particles. When non-interacting is true, then "ms" is set equal to "m", and "nu" is set equal to "mu", everywhere.

The "density" functions use the value of nu (or mu when non_interacting is true) for an initial guess. Zero is very likely a bad guess, but these functions will not warn you about this.

Definition at line 92 of file part.h.

Public Member Functions

- `part` (double `m`=0.0, double `g`=0.0)
make a particle of mass `m` and degeneracy `g`.
- virtual int `init` (double `m`, double `g`)
Set the mass `m` and degeneracy `g`.
- virtual int `anti` (`part` &`ax`)
Make an anti-particle.
- virtual const char * `type` ()
Return string denoting type ("part")

Data Fields

- double `g`
degeneracy
- double `m`
mass
- double `n`
density
- double `ed`
energy density
- double `pr`
pressure
- double `mu`

- *chemical potential*
- double [en](#)
entropy
- double [ms](#)
effective mass (Dirac unless otherwise specified)
- double [nu](#)
effective chemical potential
- bool [inc_rest_mass](#)
derivative of energy with respect to effective mass
- bool [non_interacting](#)
True if the particle is non-interacting (default true)

The documentation for this class was generated from the following file:

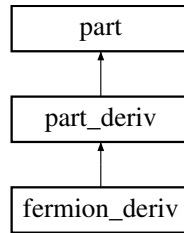
- [part.h](#)

8.39 part_deriv Class Reference

Storage for derivatives wrt μ and T .

```
#include <deriv_part.h>
```

Inheritance diagram for part_deriv:



8.39.1 Detailed Description

The variables `dndmu`, `dndT`, and `dsdT` correspond to

$$\left(\frac{dn}{d\mu}\right)_{T,V}, \quad \left(\frac{dn}{dT}\right)_{\mu,V}, \quad \text{and} \quad \left(\frac{ds}{dT}\right)_{\mu,V}$$

respectively. All other derivatives can be expressed simply in terms of these three. Note that volume derivatives are trivial, since both the entropy and number scale linearly with the volume.

Derivatives wrt to chemical potential and temperature:

There is a Maxwell relation

$$\left(\frac{ds}{d\mu}\right)_{T,V} = \left(\frac{dn}{dT}\right)_{\mu,V}$$

The pressure derivatives are trivial

$$\left(\frac{dP}{d\mu}\right)_{T,V} = n, \quad \left(\frac{dP}{dT}\right)_{\mu,V} = s$$

The energy density derivatives are related through the thermodynamic identity:

$$\left(\frac{d\varepsilon}{d\mu}\right)_{T,V} = \mu \left(\frac{dn}{d\mu}\right)_{T,V} + T \left(\frac{ds}{d\mu}\right)_{T,V}$$

$$\left(\frac{d\varepsilon}{dT}\right)_{\mu,V} = \mu \left(\frac{dn}{dT}\right)_{\mu,V} + T \left(\frac{ds}{dT}\right)_{\mu,V}$$

Other derivatives:

Note that the derivative of the entropy with respect to the temperature above is not the specific heat per particle, c_V . The specific heat per particle is

$$c_V = \frac{T}{N} \left(\frac{\partial S}{\partial T}\right)_{V,N} = \frac{T}{n} \left(\frac{\partial s}{\partial T}\right)_{V,n}$$

As noted in [Particles](#) in the User's Guide for O2scl_part, we work in units so that $\hbar = c = k_B = 1$. In this case, c_V is unitless as defined here. To compute c_V in terms of the derivatives above, note that the descendants of deriv_part provide all of the thermodynamic functions in terms of μ, V and T , so we have

$$s = s(\mu, T, V) \quad \text{and} \quad n = n(\mu, T, V).$$

We can then construct a function

$$s = s[\mu(n, T, V), T, V]$$

and then write the required derivative directly

$$\left(\frac{\partial s}{\partial T}\right)_{n,V} = \left(\frac{\partial s}{\partial \mu}\right)_{T,V} \left(\frac{\partial \mu}{\partial T}\right)_{n,V} + \left(\frac{\partial s}{\partial T}\right)_{\mu,V}.$$

Now we use the identity

$$\left(\frac{\partial \mu}{\partial T}\right)_{n,V} = - \left(\frac{\partial n}{\partial T}\right)_{\mu,V} \left(\frac{\partial n}{\partial \mu}\right)_{T,V}^{-1},$$

and the Maxwell relation above to give

$$c_V = \frac{T}{n} \left[\left(\frac{\partial s}{\partial T}\right)_{\mu,V} - \left(\frac{\partial n}{\partial T}\right)_{\mu,V}^2 \left(\frac{\partial n}{\partial \mu}\right)_{T,V}^{-1} \right]$$

which expresses the specific heat in terms of the three derivatives which are given.

For, c_P , defined as

$$c_P = \frac{T}{N} \left(\frac{\partial S}{\partial T}\right)_{N,P}$$

(which is also unitless) we can write functions

$$S = S(N, T, V) \quad \text{and} \quad V = V(N, P, T)$$

which imply

$$\left(\frac{\partial S}{\partial T}\right)_{N,P} = \left(\frac{\partial S}{\partial T}\right)_{N,V} + \left(\frac{\partial S}{\partial V}\right)_{N,T} \left(\frac{\partial V}{\partial T}\right)_{N,P}.$$

Thus we require the derivatives

$$\left(\frac{\partial S}{\partial T}\right)_{N,V}, \left(\frac{\partial S}{\partial V}\right)_{N,T}, \quad \text{and} \quad \left(\frac{\partial V}{\partial T}\right)_{N,P}.$$

To compute the new entropy derivatives, we can write

$$S = S(\mu(N, T, V), T, V)$$

to get

$$\left(\frac{\partial S}{\partial T}\right)_{N,V} = \left(\frac{\partial S}{\partial \mu}\right)_{T,V} \left(\frac{\partial \mu}{\partial T}\right)_{N,V} + \left(\frac{\partial S}{\partial T}\right)_{\mu,V},$$

and

$$\left(\frac{\partial S}{\partial V}\right)_{N,T} = \left(\frac{\partial S}{\partial \mu}\right)_{T,V} \left(\frac{\partial \mu}{\partial V}\right)_{N,T} + \left(\frac{\partial S}{\partial V}\right)_{\mu,T}.$$

These require the chemical potential derivatives which have associated Maxwell relations

$$\left(\frac{\partial \mu}{\partial T}\right)_{N,V} = -\left(\frac{\partial S}{\partial N}\right)_{T,V} \quad \text{and} \quad \left(\frac{\partial \mu}{\partial V}\right)_{N,T} = -\left(\frac{\partial P}{\partial N}\right)_{T,V}.$$

Finally, we can rewrite the derivatives on the right hand sides in terms of derivatives of functions of μ, V and T ,

$$\left(\frac{\partial S}{\partial N}\right)_{T,V} = \left(\frac{\partial S}{\partial \mu}\right)_{T,V} \left(\frac{\partial N}{\partial \mu}\right)_{T,V}^{-1},$$

and

$$\left(\frac{\partial P}{\partial N}\right)_{T,V} = \left(\frac{\partial P}{\partial \mu}\right)_{T,V} \left(\frac{\partial N}{\partial \mu}\right)_{T,V}^{-1}.$$

The volume derivative,

$$\left(\frac{\partial V}{\partial T}\right)_{N,P},$$

is related to the coefficient of thermal expansion, sometimes called α ,

$$\alpha \equiv \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_{N,P}.$$

We can rewrite the derivative

$$\left(\frac{\partial V}{\partial T}\right)_{N,P} = -\left(\frac{\partial P}{\partial T}\right)_{N,V} \left(\frac{\partial P}{\partial V}\right)_{N,T}^{-1}.$$

The first term can be computed from the Maxwell relation

$$\left(\frac{\partial P}{\partial T}\right)_{N,V} = \left(\frac{\partial S}{\partial V}\right)_{N,T},$$

where the entropy derivative was computed above. The second term (related to the inverse of the isothermal compressibility, $\kappa_T \equiv (-1/V)(\partial V/\partial P)_{T,N}$) can be computed from the function $P = P(\mu(N, V, T), V, T)$

$$\left(\frac{\partial P}{\partial V}\right)_{N,T} = \left(\frac{\partial P}{\partial \mu}\right)_{T,V} \left(\frac{\partial \mu}{\partial V}\right)_{N,T} + \left(\frac{\partial P}{\partial V}\right)_{\mu,T}$$

where the chemical potential derivative was computed above.

The results above can be collected to give

$$\left(\frac{\partial S}{\partial T}\right)_{N,P} = \left(\frac{\partial S}{\partial T}\right)_{\mu,V} + \frac{S^2}{N^2} \left(\frac{\partial N}{\partial \mu}\right)_{T,V} - \frac{2S}{N} \left(\frac{\partial N}{\partial T}\right)_{\mu,V},$$

which implies

$$c_P = \frac{T}{n} \left(\frac{\partial s}{\partial T}\right)_{\mu,V} + \frac{s^2 T}{n^3} \left(\frac{\partial n}{\partial \mu}\right)_{T,V} - \frac{2sT}{n^2} \left(\frac{\partial n}{\partial T}\right)_{\mu,V},$$

This derivation also gives the well-known relationship between the specific heats at constant volume and constant pressure,

$$c_P = c_V + \frac{T\alpha^2}{n\kappa_T}.$$

No derivative with respect to the bare mass is given, since classes cannot know how to relate the effective mass to the bare mass.

Definition at line 258 of file deriv_part.h.

Public Member Functions

- **part_deriv** (double mass=0.0, double dof=0.0)

Data Fields

- double **dndmu**
Derivative of number density with respect to chemical potential.
- double **dndT**
Derivative of number density with respect to temperature.
- double **dsdT**
Derivative of entropy density with respect to temperature.
- double **dndm**
Derivative of number density with respect to the effective mass.

The documentation for this class was generated from the following file:

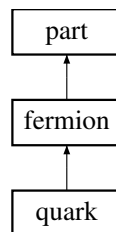
- deriv_part.h

8.40 quark Class Reference

Quark class.

```
#include <quark.h>
```

Inheritance diagram for quark:



8.40.1 Detailed Description

Definition at line 43 of file quark.h.

Public Member Functions

- **quark** (double m=0.0, double g=0.0)
Create a boson with mass m and degeneracy g .
- virtual const char * **type** ()
Return string denoting type ("quark")

Data Fields

- double **B**
Contribution to the bag constant.
- double **qq**
Quark condensate.

The documentation for this class was generated from the following file:

- quark.h

8.41 reaction_lib Class Reference

Simple reaction library.

```
#include <reaction_lib.h>
```

8.41.1 Detailed Description

This class is very experimental.

Units:

- Chapters 1,2,3, and 11: 1/s
- Chapters 4,5,6, and 7: $\text{cm}^3/\text{g}/\text{s}$
- Chapter 8 and 9: $\text{cm}^6/\text{g}^2/\text{s}$
- Chapter 10: $\text{cm}^9/\text{g}^3/\text{s}$

Chapters:

- 1: $\text{nuc1} \rightarrow \text{nuc2}$
- 2: $\text{nuc1} \rightarrow \text{nuc2} + \text{nuc3}$
- 3: $\text{nuc1} \rightarrow \text{nuc2} + \text{nuc3} + \text{nuc4}$
- 4: $\text{nuc1} + \text{nuc2} \rightarrow \text{nuc3}$
- 5: $\text{nuc1} + \text{nuc2} \rightarrow \text{nuc3} + \text{nuc4}$
- 6: $\text{nuc1} + \text{nuc2} \rightarrow \text{nuc3} + \text{nuc4} + \text{nuc5}$
- 7: $\text{nuc1} + \text{nuc2} \rightarrow \text{nuc3} + \text{nuc4} + \text{nuc5} + \text{nuc6}$
- 8: $\text{nuc1} + \text{nuc2} + \text{nuc3} \rightarrow \text{nuc4}$
- 9: $\text{nuc1} + \text{nuc2} + \text{nuc3} \rightarrow \text{nuc4} + \text{nuc5}$
- 10: $\text{nuc1} + \text{nuc2} + \text{nuc3} + \text{nuc4} \rightarrow \text{nuc5} + \text{nuc6}$
- 11: $\text{nuc1} \rightarrow \text{nuc2} + \text{nuc3} + \text{nuc4} + \text{nuc5}$

Original FORTRAN format:

```
FORMAT (i1, 4x, 6a5, 8x, a4, a1, a1, 3x, 1pe12.5)  
FORMAT (4e13.6)  
FORMAT (3e13.6)
```

Definition at line 206 of file reaction_lib.h.

Public Member Functions

- int [read_file_reacli2](#) (std::string fname)
Read from a file in the REACLIB2 format.
- int [find_in_chap](#) (std::vector< [nuclear_reaction](#) > &nrl, size_t chap, std::string nuc1, std::string nuc2="", std::string nuc3="", std::string nuc4="", std::string nuc5="", std::string nuc6="")
Find a set of nuclear reactions in a specified chapter.

Data Fields

- std::vector< [nuclear_reaction](#) > [lib](#)
The library.

Protected Member Functions

- bool [matches](#) (size_t ul, size_t ri)
Test if entry ul in the arrays matches the library reaction.

Protected Attributes

Storage for the find function

- int **fN** [6]
- int **fZ** [6]
- int **fA** [6]
- size_t **fi**

8.41.2 Member Function Documentation

8.41.2.1 int reaction_lib::read_file_reacli2 (std::string fname)

Note

This function does not check that the chapter numbers are correct for the subsequent reaction.

8.41.2.2 int reaction_lib::find_in_chap (std::vector< [nuclear_reaction](#) > &nrl, size_t chap, std::string nuc1, std::string nuc2 = " ", std::string nuc3 = " ", std::string nuc4 = " ", std::string nuc5 = " ", std::string nuc6 = " ")

The documentation for this class was generated from the following file:

- reaction_lib.h

8.42 rel_boson Class Reference

Equation of state for a relativistic boson.

```
#include <rel_boson.h>
```

8.42.1 Detailed Description

Todo Testing not completely finished.

Definition at line 48 of file rel_boson.h.

Public Member Functions

- `rel_boson ()`
Create a boson with mass m and degeneracy g .
- virtual void `calc_mu (boson &b, double temper)`
Calculate properties as function of chemical potential.
- virtual void `calc_density (boson &b, double temper)`
Calculate properties as function of density.
- virtual void `pair_mu (boson &b, double temper)`
Calculate properties with antiparticles as function of chemical potential.
- virtual void `pair_density (boson &b, double temper)`
Calculate properties with antiparticles as function of density.
- virtual void `nu_from_n (boson &b, double temper)`
Calculate effective chemical potential from density.
- void `set_inte (inte< funct > &l_nit, inte< funct > &l_dit)`
Set inte object.
- void `set_density_root (root< funct > &rp)`
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * `type ()`
Return string denoting type ("rel_boson")

Data Fields

- int `mroot_err`
The error value from mroot.
- int `inte_err`
The error value from inte.
- `cern_mroot_root< funct > def_density_root`
The default solver for calc_density().
- `gsl_inte_qagiu< funct > def_nit`
Default nondegenerate integrator.
- `gsl_inte_qag< funct > def_dit`
Default degenerate integrator.

The documentation for this class was generated from the following file:

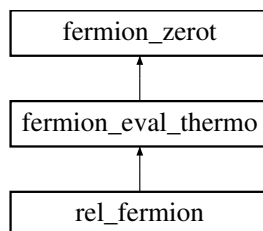
- `rel_boson.h`

8.43 rel_fermion Class Reference

Equation of state for a relativistic fermion.

```
#include <rel_fermion.h>
```

Inheritance diagram for rel_fermion:



8.43.1 Detailed Description

This class computes the thermodynamics of a relativistic fermion either as a function of the density or the chemical potential. It employs direct integration, using two different integrators for the degenerate and non-degenerate regimes. The default integrators are `gsl_inte_qag` (for degenerate fermions) and `gsl_inte_qagi` (for non-degenerate fermions). For the functions `calc_mu()` and `calc_density()`, if the temperature argument is less than or equal to zero, the functions `fermion_zerot::calc_mu_zerot()` and `fermion_zerot::calc_density_zerot()` will be used to compute the result.

Degeneracy parameter:

Define the degeneracy parameter

$$\psi = (\nu - m^*)/T$$

where ν is the effective chemical potential and m^* is the effective mass. For ψ greater than `deg_limit` (degenerate regime), a finite interval integrator is used and for ψ less than `deg_limit` (non-degenerate regime), an integrator over the interval from $[0, \infty)$ is used. In the case where `part::inc_rest_mass` is false, the degeneracy parameter is

$$\psi = (\nu + m - m^*)/T$$

Integration limits:

The upper limit on the degenerate integration is given by

$$\text{upper limit} = \sqrt{\mathcal{L}^2 - m^{*2}}$$

where $\mathcal{L} \equiv uT + \nu$ and u is `rel_fermion::upper_limit_fac`. In the case where `part::inc_rest_mass` is false, the result is

$$\text{upper limit} = \sqrt{(m + \mathcal{L})^2 - m^{*2}}$$

The entropy is only significant at the Fermi surface, thus in the degenerate case, the lower limit of the entropy integral can be given by the value of k which solves

$$-u = \frac{\sqrt{k^2 + m^{*2}} - \nu}{T}$$

The solution is

$$\text{lower limit} = \sqrt{(-uT + \nu)^2 - m^{*2}}$$

but this solution is only valid if $(m^* - \nu)/T < -u$. In the case where `part::inc_rest_mass` is false, the result is

$$\text{lower limit} = \sqrt{(-uT + m + \nu)^2 - m^{*2}}$$

which is valid if $(m^* - \nu - m)/T < -u$.

Entropy integrand:

In the degenerate regime, the entropy integrand

$$-k^2 [f \log f + (1 - f) \log (1 - f)]$$

where f is the fermionic distribution function can lose precision when $(E^* - \nu)/T$ is negative and sufficiently large in absolute magnitude. Thus when $(E^* - \nu)/T < S$ where S is stored in `deg_entropy_fac` (default is -30), the integrand is written as

$$-k^2 (E/T - \nu/T) e^{E/T - \nu/T}.$$

If $(E - \nu)/T < S$ is less than -1 times `exp_limit` (e.g. less than -200), then the entropy integrand is assumed to be zero.

Non-degenerate integrands:

The integrands in the non-degenerate regime are written in a dimensionless form, by defining $p = \sqrt{(Tu + m^*)^2 - m^{*2}}$, $\nu \equiv yT$, and $m^* \equiv mx T$. The density integrand is

$$(mx + u) \sqrt{u^2 + 2(mx)u} \left(\frac{e^y}{e^{mx+u} + e^y} \right),$$

the energy integrand is

$$(mx + u)^2 \sqrt{u^2 + 2(mx)u} \left(\frac{e^y}{e^{mx+u} + e^y} \right),$$

and the entropy integrand is

$$(mx + u) \sqrt{u^2 + 2(mx)u} (t_1 + t_2),$$

where

$$\begin{aligned} t_1 &= \log(1 + e^{y-mx-u}) / (1 + e^{y-mx-u}) \\ t_2 &= \log(1 + e^{mx+u-y}) / (1 + e^{mx+u-y}). \end{aligned}$$

Accuracy:

The default settings for for this class give an accuracy of at least 1 part in 10^6 (and frequently better than this).

When the integrators provide numerical uncertainties, these uncertainties are stored in `unc`. In the case of `calc_density()` and `pair_density()`, the uncertainty from the numerical accuracy of the solver is not included. (There is also a relatively small inaccuracy due to the mathematical evaluation of the integrands which is not included in `unc`.)

One way to improve the accuracy of the computation is just to decrease the tolerances on the default integration objects. This can be done, using, for example

```
rel_fermion rf(1.0, 2.0);
rf.def_dit.tolx/=1.0e2;
rf.def_dit.tolf/=1.0e2;
rf.def_nit.tolx/=1.0e2;
rf.def_nit.tolf/=1.0e2;
```

which decreases the both the relative and absolute tolerances for both the degenerate and non-degenerate integrators. If one is using either the `calc_density()` or `pair_density()` functions, one may also have to improve the accuracy of the solver which determines the chemical potential from the density. For the default solver, this could be done with

```
rf.def_density_root.tolx/=1.0e2;
rf.def_density_root.tolf/=1.0e2;
```

Of course if these tolerances are too small, the calculation may fail.

Todos:

Idea for Future The expressions which appear in in the integrand functions `density_fun()`, etc. could likely be improved, especially in the case where `inc_rest_mass=false`. There should not be a need to check if `ret` is finite.

Idea for Future It appears this doesn't compute the uncertainty in the chemical potential or density with `calc_density()`. This could be fixed.

Idea for Future I'd like to change the lower limit on the entropy integration, but the value in the code at the moment (stored in `ll`) makes `bm_part2.cpp` worse.

Idea for Future `pair_mu()` should set the antiparticle integrators as done in `sn_fermion`.

Definition at line 215 of file `rel_fermion.h`.

Public Member Functions

- `rel_fermion()`

Create a fermion with mass m and degeneracy g .

- virtual void `calc_mu` (fermion &f, double temper)
Calculate properties as function of chemical potential.
- virtual void `calc_density` (fermion &f, double temper)
Calculate properties as function of density.
- virtual void `pair_mu` (fermion &f, double temper)
Calculate properties with antiparticles as function of chemical potential.
- virtual void `pair_density` (fermion &f, double temper)
Calculate properties with antiparticles as function of density.
- virtual void `nu_from_n` (fermion &f, double temper)
Calculate effective chemical potential from density.
- int `set_inte` (inte< funct > &non_it, inte< funct > °_it)
Set integrators.
- int `set_density_root` (root< funct > &rp)
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * `type` ()
Return string denoting type ("rel_fermion")

Data Fields

- `fermion unc`
Storage for the uncertainty.
- `cern_mroot_root`< funct > `def_density_root`
The default solver for `calc_density()`.
- `gsl_inte_qag`< funct > `def_dit`
The default integrator for degenerate fermions.
- `gsl_inte_qagiu`< funct > `def_nit`
The default integrator for non-degenerate fermions.

Numerical parameters

- double `deg_limit`
The critical degeneracy at which to switch integration techniques (default 2)
- double `exp_limit`
The limit for exponentials to ensure integrals are finite (default 200)
- double `upper_limit_fac`
The factor for the degenerate upper limits (default 20)
- double `deg_entropy_fac`
A factor for the degenerate entropy integration (default 30)

Protected Member Functions

- double `density_fun` (double u)
The integrand for the density for non-degenerate fermions.
- double `energy_fun` (double u)
The integrand for the energy density for non-degenerate fermions.
- double `entropy_fun` (double u)
The integrand for the entropy density for non-degenerate fermions.
- double `deg_density_fun` (double u)
The integrand for the density for degenerate fermions.
- double `deg_energy_fun` (double u)
The integrand for the energy density for degenerate fermions.
- double `deg_entropy_fun` (double u)
The integrand for the entropy density for degenerate fermions.
- double `solve_fun` (double x)
Solve for the chemical potential given the density.
- double `pair_fun` (double x)
Solve for the chemical potential given the density with antiparticles.

Protected Attributes

- **inte**< **funct** > * **nit**
The non-degenerate integrator.
- **inte**< **funct** > * **dit**
The degenerate integrator.
- **root**< **funct** > * **density_root**
The solver for [calc_density\(\)](#)
- double **T**
Temperature.
- **fermion** * **fp**
Current fermion pointer.

8.43.2 Member Function Documentation

8.43.2.1 virtual void rel_fermion::calc_density (fermion & f, double temper) [virtual]

This function uses the current value of `nu` (or `mu` if the particle is non interacting) for an initial guess to solve for the chemical potential. If this guess is too small, then this function may fail.

Implements [fermion_eval_thermo](#).

The documentation for this class was generated from the following file:

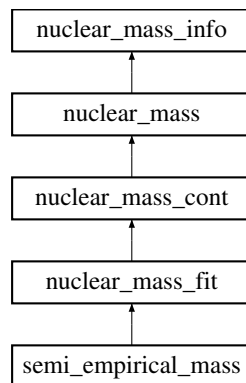
- rel_fermion.h

8.44 semi_empirical_mass Class Reference

Semi-empirical mass formula.

```
#include <nuclear_mass.h>
```

Inheritance diagram for semi_empirical_mass:



8.44.1 Detailed Description

A simple semi-empirical mass formula of the form

$$E/A = B + S_s \frac{1}{A^{1/3}} + E_c \frac{Z^2}{A^{4/3}} + S_v \left(1 - \frac{2Z}{A}\right)^2 + E_{\text{pair}}(Z, N)$$

where the pairing energy is given by

$$E_{\text{pair}}(Z, N) = -\frac{E_{\text{pair}}}{2A^{3/2}} [\cos(\pi Z) + \cos(\pi N)]$$

which is equivalent to the traditional prescription

$$E_{\text{pair}}(Z, N) = \frac{E_{\text{pair}}}{A^{3/2}} \times \begin{cases} -1 & \text{N and Z even} \\ +1 & \text{N and Z odd} \\ 0 & \text{otherwise} \end{cases}$$

when Z and N are integers.

Note

The default parameters are arbitrary, and are not determined from a fit.

There is an example of the usage of this class given in [Nuclear mass fit example](#).

Definition at line 405 of file nuclear_mass.h.

Public Member Functions

- virtual const char * [type](#) ()
Return the type, "semi_empirical_mass".
- virtual double [mass_excess_d](#) (double Z, double N)
Given Z and N, return the mass excess in MeV.
- virtual int [fit_fun](#) (size_t nv, const **ovector_base** &x)
Fix parameters from an array for fitting.
- virtual int [guess_fun](#) (size_t nv, **ovector_base** &x)
Fill array with guess from present values for fitting.

Data Fields

- double [B](#)
Binding energy (negative and in MeV, default -16)
- double [Sv](#)
Symmetry energy (in MeV, default 23.7)
- double [Ss](#)
Surface energy (in MeV, default 18)
- double [Ec](#)
Coulomb energy (in MeV, default 0.7)
- double [Epair](#)
Pairing energy (MeV, default 13.0)

The documentation for this class was generated from the following file:

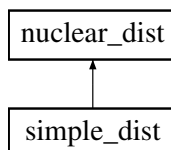
- nuclear_mass.h

8.45 simple_dist Class Reference

A simple nuclear distribution given a range in A and Z.

```
#include <nuclear_dist.h>
```

Inheritance diagram for simple_dist:



8.45.1 Detailed Description

The iterator for this distribution begins with the nucleus with the lowest Z and A, and increases A before incrementing Z and beginning again with the lowest A for that value of Z. In other words, it proceeds through all the isotopes of an element first, and then proceeds to the next element.

For example, to create a collection of isotopes of Carbon, Nitrogen and Oxygen using the most recent (2003) Atomic Mass - Evaluation, and then output the nuclei in the collection

```
ame_mass ame;
simple_dist fd(6,8,2,30,&ame);
for(nuclear_dist::iterator ndi=fd.begin();ndi!=fd.end();ndi++) {
    cout << ndi->Z << " " << ndi->A << " " << ndi->m << endl;
}
```

Idea for Future Make the vector constructor into a template so it accepts any type. Do the same for `set_dist()`.

Definition at line 165 of file `nuclear_dist.h`.

Public Member Functions

- `simple_dist()`
Create an empty distribution.
- `simple_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass &nm)`
Create a distribution from ranges in A specified for each Z.
- `simple_dist(int minZ, int maxZ, int minA, int maxA, nuclear_mass &nm)`
Create a square distribution in A and Z.
- virtual `iterator begin()`
The beginning of the distribution.
- virtual `iterator end()`
The end of the distribution.
- virtual `size_t size()`
The number of nuclei in the distribution.
- int `set_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass &nm)`
Set the distribution from ranges in A specified for each Z.
- int `set_dist(int minZ, int maxZ, int minA, int maxA, nuclear_mass &nm)`
Set a square distribution in A and Z.

8.45.2 Constructor & Destructor Documentation

8.45.2.1 `simple_dist::simple_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass & nm)`

The length of the arrays `minA` and `maxA` should be exactly $\text{maxZ} - \text{minZ} + 1$.

8.45.3 Member Function Documentation

8.45.3.1 `int simple_dist::set_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass & nm)`

The length of the arrays `minA` and `maxA` should be exactly $\text{maxZ} - \text{minZ} + 1$.

The documentation for this class was generated from the following file:

- `nuclear_dist.h`

8.46 sn_classical Class Reference

Equation of state for a classical particle with derivatives.

```
#include <sn_classical.h>
```

8.46.1 Detailed Description

Todo This does not work with `inc_rest_mass=true`

Definition at line 42 of file `sn_classical.h`.

Public Member Functions

- virtual void `calc_mu` (`part_deriv` &p, double temper)
Desc.
- virtual void `calc_density` (`part_deriv` &p, double temper)
Desc.
- virtual const char * `type` ()
Return string denoting type ("sn_classical")

The documentation for this class was generated from the following file:

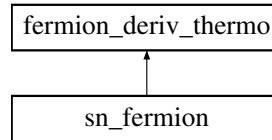
- `sn_classical.h`

8.47 sn_fermion Class Reference

Equation of state for a relativistic fermion.

```
#include <sn_fermion.h>
```

Inheritance diagram for `sn_fermion`:



8.47.1 Detailed Description

Note

This class does not work with `inc_rest_mass=true`. For example, the integration limits in `calc_mu()` need to be reworked for this case.

This implements an equation of state for a relativistic fermion using direct integration. After subtracting the rest mass from the chemical potentials, the distribution function is

$$\left\{ 1 + \exp[(\sqrt{k^2 + m^{*2}} - m - v)/T] \right\}^{-1}$$

where k is the momentum, v is the effective chemical potential, m is the rest mass, and m^* is the effective mass. For later use, we define $E^* = \sqrt{k^2 + m^{*2}}$. The degeneracy parameter is

$$\psi = (v + (m - m^*))/T$$

For ψ greater than `deg_limit` (degenerate regime), a finite interval integrator is used and for ψ less than `deg_limit` (non-degenerate regime), an integrator over the interval from $[0, \infty)$ is used. The upper limit on the degenerate integration is given by the value of the momentum k which is the solution of

$$(\sqrt{k^2 + m^{*2}} - m - v)/T = \text{flimit}$$

which is

$$\sqrt{(m + \mathcal{L})^2 - m^{*2}}$$

where $\mathcal{L} \equiv \text{flimit} \times T + v$.

For the entropy integration, we set the lower limit to

$$2\sqrt{v^2 + 2vm} - \text{upper limit}$$

since the only contribution to the entropy is at the Fermi surface.

In the non-degenerate regime, we make the substitution $u = k/T$ to ensure that the variable of integration scales properly.

Uncertainties are given in [unc](#).

Todo This needs to be corrected to calculate $\sqrt{k^2 + m^{*2}} - m$ gracefully when $m^* \approx m < k$.

Call error handler if `inc_rest_mass` is true or update to properly treat the case when `inc_rest_mass` is true.

Evaluation of the derivatives

The relevant derivatives of the distribution function are

$$\frac{\partial f}{\partial T} = f(1-f) \frac{E^* - m - v}{T^2}$$

$$\frac{\partial f}{\partial v} = f(1-f) \frac{1}{T}$$

$$\frac{\partial f}{\partial k} = -f(1-f) \frac{k}{E^* T}$$

$$\frac{\partial f}{\partial m^*} = -f(1-f) \frac{m^*}{E^* T}$$

We also need the derivative of the entropy integrand w.r.t. the distribution function, which is

$$\mathcal{S} \equiv f \ln f + (1-f) \ln(1-f) \quad \frac{\partial \mathcal{S}}{\partial f} = \ln \left(\frac{f}{1-f} \right) = \left(\frac{v - E^* + m}{T} \right)$$

where the entropy density is

$$s = -\frac{g}{2\pi^2} \int_0^\infty \mathcal{S} k^2 dk$$

The derivatives can be integrated directly ([method = direct](#)) or they may be converted to integrals over the distribution function through an integration by parts ([method = byparts](#))

$$\int_a^b f(k) \frac{dg(k)}{dk} dk = f(k)g(k)|_{k=a}^{k=b} - \int_a^b g(k) \frac{df(k)}{dk} dk$$

using the distribution function for $f(k)$ and 0 and ∞ as the limits, we have

$$\frac{g}{2\pi^2} \int_0^\infty \frac{dg(k)}{dk} f dk = \frac{g}{2\pi^2} \int_0^\infty g(k) f(1-f) \frac{k}{E^* T} dk$$

as long as $g(k)$ vanishes at $k = 0$. Rewriting,

$$\frac{g}{2\pi^2} \int_0^\infty h(k) f(1-f) dk = \frac{g}{2\pi^2} \int_0^\infty f \frac{T}{k} \left[h' E^* - \frac{h E^*}{k} + \frac{hk}{E^*} \right] dk$$

as long as $h(k)/k$ vanishes at $k = 0$.

Explicit forms

1) The derivative of the density wrt the chemical potential

$$\left(\frac{dn}{d\mu}\right)_T = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{T} f(1-f) dk$$

Using $h(k) = k^2/T$ we get

$$\left(\frac{dn}{d\mu}\right)_T = \frac{g}{2\pi^2} \int_0^\infty \left(\frac{k^2 + E^{*2}}{E^*}\right) f dk$$

2) The derivative of the density wrt the temperature

$$\left(\frac{dn}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2(E^* - m - \nu)}{T^2} f(1-f) dk$$

Using $h(k) = k^2(E^* - \nu)/T^2$ we get

$$\left(\frac{dn}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f}{T} \left[2k^2 + E^{*2} - E^*(\nu + m) - k^2 \left(\frac{\nu + m}{E^*}\right) \right] dk$$

3) The derivative of the entropy wrt the chemical potential

$$\left(\frac{ds}{d\mu}\right)_T = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - m - \nu)}{T^2} dk$$

This verifies the Maxwell relation

$$\left(\frac{ds}{d\mu}\right)_T = \left(\frac{dn}{dT}\right)_\mu$$

4) The derivative of the entropy wrt the temperature

$$\left(\frac{ds}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - m - \nu)^2}{T^3} dk$$

Using $h(k) = k^2(E^* - \nu)^2/T^3$

$$\left(\frac{ds}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f(E^* - m - \nu)}{E^* T^2} [E^{*3} + 3E^* k^2 - (E^{*2} + k^2)(\nu + m)] dk$$

5) The derivative of the density wrt the effective mass

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = -\frac{g}{2\pi^2} \int_0^\infty \frac{k^2 m^*}{E^* T} f(1-f) dk$$

Using $h(k) = -(k^2 m^*)/(E^* T)$ we get

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = -\frac{g}{2\pi^2} \int_0^\infty m^* f dk$$

Note

The dsdT integration may fail if the system is very degenerate. When method is byparts, the integral involves a large cancellation between the regions from $k \in (0, \text{ulimit}/2)$ and $k \in (\text{ulimit}/2, \text{ulimit})$. Switching to method=direct and setting the lower limit to llimit, may help, but recent testing on this gave negative values for dsdT. For very degenerate systems, an expansion may be better than trying to perform the integration. The value of the integrand at $k=0$ also looks like it might be causing difficulties.

Idea for Future It might be worth coding up direct differentiation, or differentiating the eff results, as these may succeed more generally.

Idea for Future This class will have difficulty with extremely degenerate or extremely non-degenerate systems. Fix this.

Idea for Future Create a more intelligent method for dealing with bad initial guesses for the chemical potential in `calc_density()`.

Definition at line 243 of file sn_fermion.h.

Public Member Functions

- `sn_fermion()`
Create a fermion with mass m and degeneracy g .
- virtual void `calc_mu` (`fermion_deriv` &`f`, double `temper`)
Calculate properties as function of chemical potential.
- virtual void `calc_density` (`fermion_deriv` &`f`, double `temper`)
Calculate properties as function of density.
- virtual void `pair_mu` (`fermion_deriv` &`f`, double `temper`)
Calculate properties with antiparticles as function of chemical potential.
- virtual void `pair_density` (`fermion_deriv` &`f`, double `temper`)
Calculate properties with antiparticles as function of density.
- virtual void `nu_from_n` (`fermion_deriv` &`f`, double `temper`)
Calculate effective chemical potential from density.
- void `set_inte` (`inte`< `funct` > &`unit`, `inte`< `funct` > &`udit`)
Set inte objects.
- void `set_density_root` (`root`< `funct` > &`rp`)
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * `type` ()
Return string denoting type ("sn_fermion")
- double `deriv_calibrate` (`fermion_deriv` &`f`, int `verbose`, std::string `fname`="")
Desc.

Data Fields

- double `exp_limit`
Limit of arguments of exponentials for Fermi functions (default 200.0)
- double `deg_limit`
The critical degeneracy at which to switch integration techniques (default 2.0)
- double `upper_limit_fac`
The limit for the Fermi functions (default 20.0)
- `fermion_deriv` `unc`
Storage for the most recently calculated uncertainties.
- `gsl_inte_qagiu`< `funct` > `def_nit`
The default integrator for the non-degenerate regime.
- `gsl_inte_qag`< `funct` > `def_dit`
The default integrator for the degenerate regime.
- `cern_mroot_root`< `funct` > `def_density_root`
The default solver for `npen_density()` and `pair_density()`

Method of computing derivatives

- int `method`
Method (default is `byparts`)
- static const int `direct` = 1
In the form containing $f(1-f)$.
- static const int `byparts` = 2
Integrate by parts.

8.47.2 Member Function Documentation

8.47.2.1 void `sn_fermion::set_inte` (`inte`< `funct` > &`unit`, `inte`< `funct` > &`udit`)

The first integrator is used for non-degenerate integration and should integrate from 0 to ∞ (like `gsl_inte_qagiu`). The second integrator is for the degenerate case, and should integrate between two finite values.

8.47.3 Field Documentation

8.47.3.1 double sn_fermion::upper_limit_fac

[sn_fermion](#) will ignore corrections smaller than about $\exp(-\text{flimit})$.

Definition at line 267 of file sn_fermion.h.

The documentation for this class was generated from the following file:

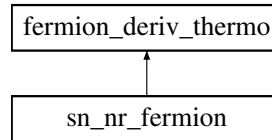
- sn_fermion.h

8.48 sn_nr_fermion Class Reference

Equation of state for a nonrelativistic fermion.

```
#include <sn_nr_fermion.h>
```

Inheritance diagram for sn_nr_fermion:



8.48.1 Detailed Description

This does not include the rest mass energy in the chemical potential or the rest mass energy density in the energy density to alleviate numerical precision problems at low densities

This implements an equation of state for a nonrelativistic fermion using direct integration. After subtracting the rest mass from the chemical potentials, the distribution function is

$$\left\{ 1 + \exp \left[\left(\frac{k^2}{2m^*} - \nu \right) / T \right] \right\}^{-1}$$

where ν is the effective chemical potential, m is the rest mass, and m^* is the effective mass. For later use, we define $E^* = k^2/2/m^*$.

Uncertainties are given in [unc](#).

Evaluation of the derivatives

The relevant derivatives of the distribution function are

$$\frac{\partial f}{\partial T} = f(1-f) \frac{E^* - \nu}{T^2}$$

$$\frac{\partial f}{\partial \nu} = f(1-f) \frac{1}{T}$$

$$\frac{\partial f}{\partial k} = -f(1-f) \frac{k}{m^* T}$$

$$\frac{\partial f}{\partial m^*} = f(1-f) \frac{k^2}{2m^{*2} T}$$

We also need the derivative of the entropy integrand w.r.t. the distribution function, which is quite simple

$$\mathcal{S} \equiv f \ln f + (1-f) \ln(1-f) \quad \frac{\partial \mathcal{S}}{\partial f} = \ln \left(\frac{f}{1-f} \right) = \left(\frac{\nu - E^*}{T} \right)$$

where the entropy density is

$$s = -\frac{g}{2\pi^2} \int_0^\infty \mathcal{S} k^2 dk$$

The derivatives can be integrated directly or they may be converted to integrals over the distribution function through an integration by parts

$$\int_a^b f(k) \frac{dg(k)}{dk} dk = f(k)g(k)|_{k=a}^{k=b} - \int_a^b g(k) \frac{df(k)}{dk} dk$$

using the distribution function for $f(k)$ and 0 and ∞ as the limits, we have

$$\frac{g}{2\pi^2} \int_0^\infty \frac{dg(k)}{dk} f dk = \frac{g}{2\pi^2} \int_0^\infty g(k) f(1-f) \frac{k}{E^* T} dk$$

as long as $g(k)$ vanishes at $k = 0$. Rewriting,

$$\frac{g}{2\pi^2} \int_0^\infty h(k) f(1-f) dk = \frac{g}{2\pi^2} \int_0^\infty f \frac{T m^*}{k} \left[h' - \frac{h}{k} \right] dk$$

as long as $h(k)/k$ vanishes at $k = 0$.

Explicit forms

1) The derivative of the density wrt the chemical potential

$$\left(\frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{T} f(1-f) dk$$

Using $h(k) = k^2/T$ we get

$$\left(\frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty m^* f dk$$

2) The derivative of the density wrt the temperature

$$\left(\frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2 (E^* - \nu)}{T^2} f(1-f) dk$$

Using $h(k) = k^2 (E^* - \nu)/T^2$ we get

$$\left(\frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f}{T} [m^* (E^* - \nu) - k^2] dk$$

3) The derivative of the entropy wrt the chemical potential

$$\left(\frac{ds}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - \nu)}{T^2} dk$$

This verifies the Maxwell relation

$$\left(\frac{ds}{d\mu} \right)_T = \left(\frac{dn}{dT} \right)_\mu$$

4) The derivative of the entropy wrt the temperature

$$\left(\frac{ds}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - \nu)^2}{T^3} dk$$

Using $h(k) = k^2 (E^* - \nu)^2/T^3$

$$\left(\frac{ds}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty f \frac{m^*}{T^2} \left[(E^* - \nu)^2 + \frac{2k^2}{m^*} (E^* - \nu) \right] dk$$

5) The derivative of the density wrt the effective mass

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{2m^{*2}T} f(1-f) k^2 dk$$

Using $h(k) = k^4/(2m^{*2}T)$ we get

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = \frac{g}{2\pi^2} \int_0^\infty f \frac{3k^2}{2m^*} dk$$

New section

$u = k^2/2/m^*/T$ and $y = \mu/T$, so

$$kdk = m^*T du$$

or

$$dk = \frac{m^*T}{\sqrt{2m^*Tu}} du = \sqrt{\frac{m^*T}{2u}} du$$

1) The derivative of the density wrt the chemical potential

$$\left(\frac{dn}{d\mu}\right)_T = \frac{gm^{*3/2}\sqrt{T}}{2^{3/2}\pi^2} \int_0^\infty u^{-1/2} f du$$

2) The derivative of the density wrt the temperature

$$\left(\frac{dn}{dT}\right)_\mu = \frac{gm^{*3/2}\sqrt{T}}{2^{3/2}\pi^2} \int_0^\infty f du \left[3u^{1/2} - yu^{-1/2}\right]$$

4) The derivative of the entropy wrt the temperature

$$\left(\frac{ds}{dT}\right)_\mu = \frac{gm^{*3/2}T^{1/2}}{2^{3/2}\pi^2} \int_0^\infty f \left[5u^{3/2} - 6yu^{1/2} + y^2u^{-1/2}\right] du$$

5) The derivative of the density wrt the effective mass

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = \frac{3gm^{*1}/2T^{3/2}}{2^{3/2}\pi^2} \int_0^\infty u^{1/2} f du$$

Definition at line 221 of file sn_nr_fermion.h.

Public Member Functions

- [sn_nr_fermion\(\)](#)
Create a fermion with mass m and degeneracy g .
- virtual void [calc_mu](#) ([fermion_deriv](#) & f , double temper)
Calculate properties as function of chemical potential.
- virtual void [calc_density](#) ([fermion_deriv](#) & f , double temper)
Calculate properties as function of density.
- virtual void [pair_mu](#) ([fermion_deriv](#) & f , double temper)
Calculate properties with antiparticles as function of chemical potential.
- virtual void [pair_density](#) ([fermion_deriv](#) & f , double temper)
Calculate properties with antiparticles as function of density.
- virtual void [nu_from_n](#) ([fermion_deriv](#) & f , double temper)
Calculate effective chemical potential from density.
- void [set_density_root](#) ([root](#)< [funct](#) > & rp)
Set the solver for use in calculating the chemical potential from the density.
- virtual const char * [type](#) ()
Return string denoting type ("sn_nr_fermion")

Data Fields

- double `flimit`
The limit for the Fermi functions (default 20.0)
- `fermion_deriv_unc`
Storage for the most recently calculated uncertainties.
- bool `guess_from_nu`
If true, use the present value of the chemical potential as a guess for the new chemical potential.
- `cern_mroot_root` < `funct` > `def_density_root`
The default solver for `npen_density()` and `pair_density()`

Protected Member Functions

- double `solve_fun` (double x)
Function to compute chemical potential from density.
- double `pair_fun` (double x)
Function to compute chemical potential from density when antiparticles are included.

Protected Attributes

- double `T`
Desc.
- `fermion_deriv` * `fp`
Desc.
- `root` < `funct` > * `density_root`
Solver to compute chemical potential from density.

8.48.2 Field Documentation

8.48.2.1 double `sn_nr_fermion::flimit`

`sn_nr_fermion` will ignore corrections smaller than about $\exp(-\text{flimit})$.

Definition at line 234 of file `sn_nr_fermion.h`.

The documentation for this class was generated from the following file:

- `sn_nr_fermion.h`

8.49 thermo Class Reference

A class for the thermodynamical variables (energy density, pressure, entropy density)

```
#include <part.h>
```

8.49.1 Detailed Description

Definition at line 45 of file `part.h`.

Public Member Functions

- const char * `type` ()
Return string denoting type ("thermo")

Data Fields

- double `pr`
pressure
- double `ed`
energy density
- double `en`
entropy density

The documentation for this class was generated from the following file:

- [part.h](#)

9 File Documentation

9.1 hdf_nucmass_io.h File Reference

File for HDF input of the O₂scl [ame_mass](#) and [mnmsk_mass](#) data files.

```
#include <hdf5.h> #include <hdf5_hl.h> #include <o2scl/constants.h> #include <o2scl/hdf-
_file.h> #include <o2scl/lib_settings.h> #include <o2scl/nuclear_mass.h>
```

9.1.1 Detailed Description

Definition in file [hdf_nucmass_io.h](#).

Functions

- int [ame_load](#) ([ame_mass](#) &ame, std::string version, string dir="")
Read data for [ame_mass](#) from an HDF table.
- int [mnmsk_load](#) ([mnmsk_mass](#) &mnmsk, string dir="")
Read data for [mnmsk_mass](#) from an HDF table.
- int [hfb_load](#) ([hfb_mass](#) &hfb, int model=14, string dir="")
Read data for [hfb_mass](#) from an HDF table.
- int [hfb_sp_load](#) ([hfb_sp_mass](#) &hfb, int model=21, string dir="")
Read data for [hfb_sp_mass](#) from an HDF table.
- int [ame_load](#) (o2scl::ame_mass &ame, std::string version, std::string dir="")
- int [mnmsk_load](#) (o2scl::mnmsk_mass &mnmsk, std::string dir="")
- int [hfb_load](#) (o2scl::hfb_mass &hfb, int model=14, std::string dir="")
- int [hfb_sp_load](#) (o2scl::hfb_sp_mass &hfb, int model=21, std::string dir="")

9.1.2 Function Documentation

9.1.2.1 int hfb_load (hfb_mass & hfb, int model = 14, string dir = " ")

Valid values of `model` at present are 2, 8, and 14, corresponding to the HFB2 ([Goriely02](#)), HFB8 ([Samyn04](#)), and HFB14 ([Goriely07](#)). If a number other than these three is given, the error handler is called.

9.1.2.2 int hfb_sp_load (hfb_sp_mass & hfb, int model = 21, string dir = " ")

Valid values of `model` at present are 17 and 21, corresponding to the HFB17 ([Goriely02](#)) and HFB21 ([Samyn04](#)). If a number other than these two is given, the error handler is called.

9.2 part.h File Reference

File for definitions for [thermo](#) and [part](#).

```
#include <string> #include <iostream> #include <cmath> #include <o2scl/constants.h> #include  
<o2scl/inte.h> #include <o2scl/funct.h> #include <o2scl/mroot.h>
```

9.2.1 Detailed Description

Definition in file [part.h](#).

Data Structures

- class [thermo](#)
A class for the thermodynamical variables (energy density, pressure, entropy density)
- class [part](#)
Particle base class.

Functions

- [thermo operator+](#) (const [thermo](#) &left, const [thermo](#) &right)
Addition operator.
- [thermo operator-](#) (const [thermo](#) &left, const [thermo](#) &right)
Subtraction operator.
- [thermo operator+](#) (const [thermo](#) &left, const [part](#) &right)
Addition operator.
- [thermo operator-](#) (const [thermo](#) &left, const [part](#) &right)
Subtraction operator.