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<< Algebra'Horner'
```

- **For Bosons (try 2):**
- **Non-degenerate and non-relativstic:**

```
pcoeff = t^2 Exp[n (ψ + 1 / t)] / n^2
```

$$\frac{e^{n\left(\frac{1}{t}+\psi\right)}\,t^2}{n^2}$$

```
besselseries = Simplify[Normal[Series[BesselK[2, x], {x, ∞, 6}]]]
```

$$\frac{1}{4194304}\,e^{-x}\sqrt{\frac{\pi}{2}}\left(\frac{1}{x}\right)^{13/2}\left(4729725-2162160\,x+1330560\,x^2-1290240\,x^3+3440640\,x^4+7864320\,x^5+4194304\,x^6\right)$$

```
pn = Simplify[pcoeff (besselseries /. x → n / t)]
```

$$\frac{1}{4194304\,n^6}\,e^{n\psi}\sqrt{\frac{\pi}{2}}\left(\frac{t}{n}\right)^{5/2}\left(4194304\,n^6+7864320\,n^5\,t+3440640\,n^4\,t^2-1290240\,n^3\,t^3+1330560\,n^2\,t^4-2162160\,n\,t^5+4729725\,t^6\right)$$

Separate out the first term to make the expansion more clear:

```
firstterm = (Simplify[pn / t^(5/2), t > 0] /. t → 0) t^(5/2)
```

$$e^{n\psi}\left(\frac{1}{n}\right)^{5/2}\sqrt{\frac{\pi}{2}}\,t^{5/2}$$

```
Expand[Simplify[pn / firstterm, n > 0]]
```

$$1+\frac{15\,t}{8\,n}+\frac{105\,t^2}{128\,n^2}-\frac{315\,t^3}{1024\,n^3}+\frac{10395\,t^4}{32768\,n^4}-\frac{135135\,t^5}{262144\,n^5}+\frac{4729725\,t^6}{4194304\,n^6}$$

- **Non-degenerate and extremely relativistic:**

```
pcoeff = t^2 Exp[n ψ] / n^2
```

$$\frac{e^{n\psi}\,t^2}{n^2}$$

```
besselseries = Simplify[Normal[Series[BesselK[2, x] Exp[x], {x, 0, 1}]]]
```

$$\frac{1}{2}+\frac{2}{x^2}+\frac{2}{x}-\frac{x}{6}$$

```
pn = Simplify[pcoeff (besselseries /. x → n / t)]
```

$$\frac{e^{n\psi}\,t\left(-n^3+3\,n^2\,t+12\,n\,t^2+12\,t^3\right)}{6\,n^4}$$

```
Expand[pn n^4 / 2 / t^4 / Exp[n ψ]]
```

$$1-\frac{n^3}{12\,t^3}+\frac{n^2}{4\,t^2}+\frac{n}{t}$$

Extremely- degenerate:

```
piand = 1 / 3 l^4 / Sqrt[l^2 + 1] / (-1 + Exp[(Sqrt[l^2 + 1] - 1) / t - ψ])
```

$$\frac{l^4}{3\left(-1+e^{\frac{-1+\sqrt{1+l^2}}{t}-\psi}\right)\sqrt{1+l^2}}$$

```
lsub = Sqrt[(y t + 1)^2 - 1]
```

$$\sqrt{-1+(1+t\,y)^2}$$

Calculate dldy:

```
dldy = D[lsub, y]
```

$$\frac{t\,(1+t\,y)}{\sqrt{-1+(1+t\,y)^2}}$$

```
p2 = Simplify[piand /. l → Sqrt[(En + 1)^2 - 1], En > 0]
```

$$\frac{e^{\psi}\,En^2\,(2+En)^2}{3\left(e^{En/t}-e^{\psi}\right)(1+En)}$$

```
p3 = Simplify[3 p2 dldy (Exp[y] - Exp[ψ]) / Exp[ψ] / t^(5/2) / y^(3/2) / 2^(3/2) /. En → y t]
```

$$\frac{(t\,y\,(2+t\,y))^{3/2}}{2\sqrt{2}\,t^{3/2}\,y^{3/2}}$$

```
Series[p3, {t, 0, 6}]
```

$$1+\frac{3\,y\,t}{4}+\frac{3\,y^2\,t^2}{32}-\frac{y^3\,t^3}{128}+\frac{3\,y^4\,t^4}{2048}-\frac{3\,y^5\,t^5}{8192}+\frac{7\,y^6\,t^6}{65\,536}+O[t]^{13/2}$$

```
px = Series[p2 /. En → y t, {t, 0, 5}]
```

$$\frac{4\,e^{\psi}\,y^2\,t^2}{3\left(e^{y}-e^{\psi}\right)}+\frac{e^{\psi}\,y^4\,t^4}{3\left(e^{y}-e^{\psi}\right)}-\frac{\left(e^{\psi}\,y^5\right)t^5}{3\left(e^{y}-e^{\psi}\right)}+O[t]^6$$

```
px2 = Integrate[Normal[px], {y, 0, ∞}]
```

$$\frac{8}{3}\,t^2\,\mathrm{PolyLog}\left[3,\,e^{\psi}\right]+8\,t^4\left(\mathrm{PolyLog}\left[5,\,e^{\psi}\right]-5\,t\,\mathrm{PolyLog}\left[6,\,e^{\psi}\right]\right)$$

```
Series[px2, {ψ, 0, 3}]
```

$$\left(-\frac{8}{189}\pi^6\,t^5+\frac{8}{3}\,t^2\,\mathrm{Zeta}[3]+8\,t^4\,\mathrm{Zeta}[5]\right)+\left(\frac{4\,\pi^2\,t^2}{9}+\frac{4\,\pi^4\,t^4}{45}-40\,t^5\,\mathrm{Zeta}[5]\right)\psi+$$
$$\left(2\,t^2-\frac{4}{3}\,i\,\pi\,t^2-\frac{2\,\pi^4\,t^5}{9}-\frac{4}{3}\,t^2\,\mathrm{Log}[\psi]+4\,t^4\,\mathrm{Zeta}[3]\right)\psi^2+\left(-\frac{2\,t^2}{9}+\frac{2\,\pi^2\,t^4}{9}-\frac{20}{3}\,t^5\,\mathrm{Zeta}[3]\right)\psi^3+O[\psi]^4$$

ED:

```
px3 = Simplify[Normal[Series[Normal[Series[p2 /. En → y t, {t, ∞, 3}]], {ψ, 0, 3}]]]
```

$$\frac{1}{18\,(-1+e^{y})^{4}\,t^{3}\,y^{3}}\left(1-t\,y+t^{2}\,y^{2}-t^{3}\,y^{3}+t^{4}\,y^{4}+3\,t^{5}\,y^{5}+t^{6}\,y^{6}\right)$$
$$\left(-6+e^{y}\left(18+6\,\psi-3\,\psi^{2}+\psi^{3}\right)+e^{3\,y}\left(6+6\,\psi+3\,\psi^{2}+\psi^{3}\right)+2\,e^{2\,y}\left(-9-6\,\psi+2\,\psi^{3}\right)\right)$$