

In[1]:= Off[General::"spell"]; Off[General::"spell1"];

■ The Hamiltonian

In[2]:= h2mns = 1/2/M + n/4 (t1 (1 + x1/2) + t2 (1 + x2/2)) + nn/4 (t2 (1/2 + x2) - t1 (1/2 + x1))

Out[2]= $\frac{1}{2M} + \frac{1}{4} n \left(t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right) + \frac{1}{4} nn \left(-t_1 \left(\frac{1}{2} + x_1 \right) + t_2 \left(\frac{1}{2} + x_2 \right) \right)$

In[3]:= h2mps = 1/2/M + n/4 (t1 (1 + x1/2) + t2 (1 + x2/2)) + np/4 (t2 (1/2 + x2) - t1 (1/2 + x1))

Out[3]= $\frac{1}{2M} + \frac{1}{4} n \left(t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right) + \frac{1}{4} np \left(-t_1 \left(\frac{1}{2} + x_1 \right) + t_2 \left(\frac{1}{2} + x_2 \right) \right)$

In[4]:= kfn = (3 π² nn)¹/³

Out[4]= $3^{1/3} nn^{1/3} \pi^{2/3}$

In[5]:= kfp = (3 π² np)¹/³

Out[5]= $3^{1/3} np^{1/3} \pi^{2/3}$

In[6]:= τn = kfn⁵ / 5 / π²

Out[6]= $\frac{3}{5} 3^{2/3} nn^{5/3} \pi^{4/3}$

In[7]:= τp = kfp⁵ / 5 / π²

Out[7]= $\frac{3}{5} 3^{2/3} np^{5/3} \pi^{4/3}$

In[8]:= Hbulk = Simplify[τn h2mns + τp h2mps + t0/2 ((1 + x0/2) n² - (1/2 + x0) (nn² + np²)) +
a t3/6 ((1 + x3/2) n^α nn np + 2^{α-2} (1 - x3) (nn^{α+2} + np^{α+2})) +
b t3/12 ((1 + x3/2) n² - (1/2 + x3) (nn² + np²)) n^α /. n → nn + np]

Out[8]= $\frac{1}{120} \left(60 t_0 \left(- (nn^2 + np^2) \left(\frac{1}{2} + x_0 \right) + \frac{1}{2} (nn + np)^2 (2 + x_0) \right) - \right.$
 $\frac{9 3^{2/3} np^{5/3} \pi^{4/3} (-4 - M (np (t_1 - t_1 x_1 + 3 t_2 (1 + x_2)) + nn (t_1 (2 + x_1) + t_2 (2 + x_2))))}{M} -$
 $\frac{9 3^{2/3} nn^{5/3} \pi^{4/3} (-4 + M (nn (t_1 (-1 + x_1) - 3 t_2 (1 + x_2)) - np (t_1 (2 + x_1) + t_2 (2 + x_2))))}{M} -$
 $5 b (nn + np)^\alpha t_3 (nn^2 (-1 + x_3) + np^2 (-1 + x_3) - 2 nn np (2 + x_3)) +$
 $5 a t_3 (-2^\alpha (nn^{2+\alpha} + np^{2+\alpha}) (-1 + x_3) + 2 nn np (nn + np)^\alpha (2 + x_3)) \left. \right)$

■ The energy of symmetric nuclear matter:

In[9]:= Hnuc = Simplify[Hbulk /. nn → n/2 /. np → n/2]

Out[9]= $\frac{n^{5/3} \left(10 M \left(6 n^{1/3} t_0 + (a + b) n^{\frac{1}{3}+\alpha} t_3 \right) + 3 2^{1/3} 3^{2/3} \pi^{4/3} (8 + M n (3 t_1 + 5 t_2 + 4 t_2 x_2)) \right)}{160 M}$

$$\text{In}[10]:= \text{kr23} = 3 / 10 / \text{M} (3 / 2 \pi^2 n)^{2/3}$$

$$\beta = \text{M} / 2 (1 / 4 (3 t1 + 5 t2) + t2 x2)$$

$$\text{Out}[10]= \frac{3 \left(\frac{3}{2}\right)^{2/3} n^{2/3} \pi^{4/3}}{10 \text{M}}$$

$$\text{Out}[11]= \frac{1}{2} \text{M} \left(\frac{1}{4} (3 t1 + 5 t2) + t2 x2 \right)$$

$$\text{In}[12]:= \text{Hnuc2} = n (\text{kr23} (1 + \beta n) + 3 t0 n / 8 + (a + b) t3 n^{1+\alpha} / 16)$$

$$\text{Out}[12]= n \left(\frac{3 n t0}{8} + \frac{1}{16} (a + b) n^{1+\alpha} t3 + \frac{3 \left(\frac{3}{2}\right)^{2/3} n^{2/3} \pi^{4/3} \left(1 + \frac{1}{2} \text{M} n \left(\frac{1}{4} (3 t1 + 5 t2) + t2 x2\right)\right)}{10 \text{M}} \right)$$

$$\text{In}[13]:= \text{Simplify}[\text{Hnuc2} - \text{Hnuc}]$$

$$\text{Out}[13]= 0$$

■ The effective masses in symmetric nuclear matter:

The inverse of the reduced effective mass:

$$\text{In}[14]:= \text{MoMs} = \text{Simplify}[2 \text{M} (\text{h2mns} /. \text{nn} \rightarrow n / 2 /. \text{np} \rightarrow n / 2)]$$

$$\text{Out}[14]= \frac{1}{8} (8 + \text{M} n (3 t1 + t2 (5 + 4 x2)))$$

$$\text{In}[15]:= \text{MoMs2} = 1 + \beta n$$

$$\text{Out}[15]= 1 + \frac{1}{2} \text{M} n \left(\frac{1}{4} (3 t1 + 5 t2) + t2 x2 \right)$$

$$\text{In}[16]:= \text{Simplify}[\text{MoMs} - \text{MoMs2}]$$

$$\text{Out}[16]= 0$$

■ The incompressibility of nuclear matter:

$$\text{In}[17]:= \text{K} = \text{Simplify}[9 n^2 \text{D}[\text{D}[(\text{Hbulk} /. \text{nn} \rightarrow n / 2 /. \text{np} \rightarrow n / 2) / n, n], n]]$$

$$\text{Out}[17]= \frac{3 n^{2/3} \left(2^{1/3} 3^{2/3} \pi^{4/3} (-8 + 5 \text{M} n (3 t1 + t2 (5 + 4 x2))) + 15 (a + b) \text{M} n^{\frac{1}{3}+\alpha} t3 \alpha (1 + \alpha)\right)}{80 \text{M}}$$

$$\text{In}[18]:= \text{K2} = -2 \text{kr23} + 10 \text{kr23} \beta n + 9 / 16 \alpha (\alpha + 1) (a + b) t3 n^{1+\alpha}$$

$$\text{Out}[18]= -\frac{3 \left(\frac{3}{2}\right)^{2/3} n^{2/3} \pi^{4/3}}{5 \text{M}} + \frac{3}{2} \left(\frac{3}{2}\right)^{2/3} n^{5/3} \pi^{4/3} \left(\frac{1}{4} (3 t1 + 5 t2) + t2 x2\right) + \frac{9}{16} (a + b) n^{1+\alpha} t3 \alpha (1 + \alpha)$$

$$\text{In}[19]:= \text{Simplify}[\text{K2} - \text{K}]$$

$$\text{Out}[19]= 0$$

■ The symmetry energy:

Neutron matter:

In[20]:= **Solve**[{(nn - np) == n δ, n == nn + np}, {nn, np}]

Out[20]= $\left\{ \left\{ nn \rightarrow -\frac{1}{2} (-n - n \delta), np \rightarrow -\frac{1}{2} (-n + n \delta) \right\} \right\}$

In[21]:= **Esym** =
Simplify[(D[D[(Hbulk /. nn → 1/2 (n + n δ) /. np → 1/2 (n - n δ)], δ], δ] / 2 / n) /. δ → 0]

Out[21]= $\frac{1}{96 M} (8 2^{1/3} 3^{2/3} n^{2/3} \pi^{4/3} - 12 M n t_0 (1 + 2 x_0) + 2 2^{1/3} 3^{2/3} M n^{5/3} \pi^{4/3} (-3 t_1 x_1 + t_2 (4 + 5 x_2)) - M n^{1+\alpha} t_3 (2 b (1 + 2 x_3) + a (2 - 3 \alpha - \alpha^2 + x_3 (4 + 3 \alpha + \alpha^2))))$

In[22]:= **Esym2** = 5 / 9 kr23 + 10 / 3 Mkr23 n (1 / 6 t2 (1 + 5 / 4 x2) - 1 / 8 t1 x1) - 1 / 24 b t3 (1 / 2 + x3) n^{1+α} - 1 / 4 t0 (1 / 2 + x0) n - $\frac{1}{96} a n^{1+\alpha} t_3 (2 - \alpha (3 + \alpha) + x_3 (4 + \alpha (3 + \alpha)))$

Out[22]= $\frac{n^{2/3} \pi^{4/3}}{2 2^{2/3} 3^{1/3} M} - \frac{1}{4} n t_0 \left(\frac{1}{2} + x_0 \right) + \left(\frac{3}{2} \right)^{2/3} n^{5/3} \pi^{4/3} \left(-\frac{t_1 x_1}{8} + \frac{1}{6} t_2 \left(1 + \frac{5 x_2}{4} \right) \right) - \frac{1}{24} b n^{1+\alpha} t_3 \left(\frac{1}{2} + x_3 \right) - \frac{1}{96} a n^{1+\alpha} t_3 (2 - \alpha (3 + \alpha) + x_3 (4 + \alpha (3 + \alpha)))$

In[23]:= **Simplify**[Esym - Esym2]

Out[23]= 0

In[24]:= **EPaul** = 5 / 9 kr23 (1 + 6 M n (1 / 6 t2 (1 + 5 / 4 x2) - t1 x1 / 8)) - t0 / 4 (1 / 2 + x0) n + a t3 / 24 n^{1+α} (-1 - x3 / 2 + 1 / 4 (1 - x3) (α + 2) (α + 1)) - b t3 / 24 (1 / 2 + x3) n^{1+α}

Out[24]= $-\frac{1}{4} n t_0 \left(\frac{1}{2} + x_0 \right) + \frac{n^{2/3} \pi^{4/3} (1 + 6 M n (-\frac{t_1 x_1}{8} + \frac{1}{6} t_2 (1 + \frac{5 x_2}{4})))}{2 2^{2/3} 3^{1/3} M} - \frac{1}{24} b n^{1+\alpha} t_3 \left(\frac{1}{2} + x_3 \right) + \frac{1}{24} a n^{1+\alpha} t_3 \left(-1 - \frac{x_3}{2} + \frac{1}{4} (1 - x_3) (1 + \alpha) (2 + \alpha) \right)$

In[25]:= **Simplify**[Esym - EPaul]

Out[25]= 0

■ Re-express Hamiltonian as in code:

In[26]:= **vars** = {ham1 -> t0 / 2 (1 + x0 / 2),
ham2 -> -t0 / 2 (1 / 2 + x0),
ham3 -> a t3 / 6 (1 + x3 / 2),
ham4 -> a t3 2^{α-2} / 6 (1 - x3),
ham5 -> b t3 / 12 (1 + x3 / 2),
ham6 -> -b t3 / 12 (1 / 2 + x3)}

Out[26]= {ham1 → $\frac{1}{2} t_0 \left(1 + \frac{x_0}{2} \right)$, ham2 → $-\frac{1}{2} t_0 \left(\frac{1}{2} + x_0 \right)$, ham3 → $\frac{1}{6} a t_3 \left(1 + \frac{x_3}{2} \right)$,
ham4 → $\frac{1}{3} 2^{-3+\alpha} a t_3 (1 - x_3)$, ham5 → $\frac{1}{12} b t_3 \left(1 + \frac{x_3}{2} \right)$, ham6 → $-\frac{1}{12} b t_3 \left(\frac{1}{2} + x_3 \right)$ }

In[27]:= **Hcode** = **tn** h2mns + **rp** h2mps + ham1 n² + ham2 (nn² + np²) + ham3 n^α nn np + ham4 (nn^{α+2} + np^{α+2}) + ham5 n^{2+α} + ham6 (nn² + np²) n^α

Out[27]= ham1 n² + ham5 n^{2+α} + ham3 n^α nn np + ham2 (nn² + np²) + ham6 n^α (nn² + np²) + ham4 (nn^{2+α} + np^{2+α}) + $\frac{3}{5} 3^{2/3} n n^{5/3} \pi^{4/3} \left(\frac{1}{2 M} + \frac{1}{4} n \left(t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right) + \frac{1}{4} n n \left(-t_1 \left(\frac{1}{2} + x_1 \right) + t_2 \left(\frac{1}{2} + x_2 \right) \right) \right) + \frac{3}{5} 3^{2/3} n p^{5/3} \pi^{4/3} \left(\frac{1}{2 M} + \frac{1}{4} n \left(t_1 \left(1 + \frac{x_1}{2} \right) + t_2 \left(1 + \frac{x_2}{2} \right) \right) + \frac{1}{4} n p \left(-t_1 \left(\frac{1}{2} + x_1 \right) + t_2 \left(\frac{1}{2} + x_2 \right) \right) \right)$

In[28]:= **Simplify**[(Hcode /. vars /. n → nn + np) - Hbulk]

Out[28]= 0

In[29]:= **D**[(Hcode /. n → nn + np), nn]

Out[29]= $2 \text{ham2} \text{nn} + 2 \text{ham1} (\text{nn} + \text{np}) + 2 \text{ham6} \text{nn} (\text{nn} + \text{np})^\alpha +$
 $\text{ham3} \text{np} (\text{nn} + \text{np})^\alpha + \frac{3}{20} 3^{2/3} \text{np}^{5/3} \pi^{4/3} \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) +$
 $\frac{3}{5} 3^{2/3} \text{nn}^{5/3} \pi^{4/3} \left(\frac{1}{4} \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) + \frac{1}{4} \left(-\text{t1} \left(\frac{1}{2} + \text{x1} \right) + \text{t2} \left(\frac{1}{2} + \text{x2} \right) \right) \right) + 3^{2/3} \text{nn}^{2/3}$
 $\pi^{4/3} \left(\frac{1}{2M} + \frac{1}{4} (\text{nn} + \text{np}) \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) + \frac{1}{4} \text{nn} \left(-\text{t1} \left(\frac{1}{2} + \text{x1} \right) + \text{t2} \left(\frac{1}{2} + \text{x2} \right) \right) \right) +$
 $\text{ham3} \text{nn} \text{np} (\text{nn} + \text{np})^{-1+\alpha} \alpha + \text{ham6} (\text{nn} + \text{np})^{-1+\alpha} (\text{nn}^2 + \text{np}^2) \alpha +$
 $\text{ham4} \text{nn}^{1+\alpha} (2 + \alpha) + \text{ham5} (\text{nn} + \text{np})^{1+\alpha} (2 + \alpha)$

In[30]:= **D**[(Hcode /. n → nn + np), np]

Out[30]= $2 \text{ham2} \text{np} + 2 \text{ham1} (\text{nn} + \text{np}) + \text{ham3} \text{nn} (\text{nn} + \text{np})^\alpha +$
 $2 \text{ham6} \text{np} (\text{nn} + \text{np})^\alpha + \frac{3}{20} 3^{2/3} \text{nn}^{5/3} \pi^{4/3} \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) +$
 $\frac{3}{5} 3^{2/3} \text{np}^{5/3} \pi^{4/3} \left(\frac{1}{4} \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) + \frac{1}{4} \left(-\text{t1} \left(\frac{1}{2} + \text{x1} \right) + \text{t2} \left(\frac{1}{2} + \text{x2} \right) \right) \right) + 3^{2/3} \text{np}^{2/3}$
 $\pi^{4/3} \left(\frac{1}{2M} + \frac{1}{4} (\text{nn} + \text{np}) \left(\text{t1} \left(1 + \frac{\text{x1}}{2} \right) + \text{t2} \left(1 + \frac{\text{x2}}{2} \right) \right) + \frac{1}{4} \text{np} \left(-\text{t1} \left(\frac{1}{2} + \text{x1} \right) + \text{t2} \left(\frac{1}{2} + \text{x2} \right) \right) \right) +$
 $\text{ham3} \text{nn} \text{np} (\text{nn} + \text{np})^{-1+\alpha} \alpha + \text{ham6} (\text{nn} + \text{np})^{-1+\alpha} (\text{nn}^2 + \text{np}^2) \alpha +$
 $\text{ham4} \text{np}^{1+\alpha} (2 + \alpha) + \text{ham5} (\text{nn} + \text{np})^{1+\alpha} (2 + \alpha)$

In[31]:=