

# O<sub>2</sub>scl\_part - Particle Sub-Library for O<sub>2</sub>scl

Version 0.805

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# 1 Main Page

## 1.1 User's guide

- [Particles](#)
  - [Atomic nuclei](#)
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- 

## 1.2 Particles

These classes in the library `o2scl_part` calculate the thermodynamic properties of interacting and non-interacting quantum and [classical](#) particles.

The class [part](#) is the basic structure for a particle:

- `m` - mass
- `g` - degeneracy factor (e.g.  $2j + 1$ )
- `n` - number density
- `ns` - scalar number density
- `ed` - energy density
- `pr` - pressure
- `en` - entropy
- `ms` - effective mass
- `nu` - effective chemical potential
- `bool inc_rest_mass` - True if the rest mass is included
- `bool non_interacting` - False if the particle includes interactions
- `std::string name` - a 'name' field to distinguish different particles of the same class

The data members [part::ms](#) and [part::nu](#) allow one to specify modifications to the mass and the chemical potential due to interactions. This allows one to calculate the properties of particle due to interactions so long as the basic form of the free-particle dispersion relation is unchanged, i.e.

$$\sqrt{k^2 + m^2} - \mu \rightarrow \sqrt{k^2 + m^{*2}} - \nu$$

Typically, if the particle is non-interacting, then `mu` and `m` are copied to `nu` and `ms`, computations are performed with `nu` and `ms`, and then, if necessary, the result for `nu` is copied back to `mu`.

If [part::inc\\_rest\\_mass](#) is `true` (as is the default in all of the classes except [nucleus](#)), then all functions include the rest mass energy density in the energy density, the "mu" functions expect that the rest mass is included in [part::mu](#) or [part::nu](#) as input and the "density" functions output [part::mu](#) or [part::nu](#) including the rest mass.

When `inc_rest_mass` is true, antiparticles are implemented by choosing the antiparticle chemical potential to be  $-\mu$ , and when `inc_rest_mass` is false, antiparticles are implemented by choosing the chemical potential of the antiparticles to be  $-\mu - 2m$ .

The thermodynamic identity used to compute the pressure for interacting particles is

$$P = -\epsilon + sT + \nu n$$


---

where `nu` is used. This way, the particle class doesn't need to know about the structure of the interactions to ensure that the thermodynamic identity is satisfied. Note that in the `o2scl_eos` library, where in the equations of state the normal thermodynamic identity is used

$$P = -\varepsilon + sT + \mu n$$

Frequently, the interactions which create an effective chemical potential which is different than `mu` thus create extra terms in the pressure and the energy density for the given equation of state.

At zero temperature, fermions and bosons can be treated exactly in the classes `fermion` and `boson`. `quark` is a descendant of the `fermion` class which contains extra data members for the `quark` condensate and the contribution to the bag constant. `classical` is a descendant of both `fermion` and `boson` and calculates everything in the `classical` limit.

At finite temperature, there are different classes corresponding to different approaches to computing the integrals over the distribution functions. The approximation scheme from [Johns96](#) is used in `eff_boson`, `eff_fermion`, and `eff_quark`. An exact method is used in `rel_boson` and `rel_fermion`, but these are necessarily quite a bit slower.

The class `nonrel_fermion` assumes a non-relativistic dispersion relation for fermions. It includes zero-temperature methods and an exact method for finite temperatures. The non-relativistic integrands are much simpler and `nonrel_fermion` uses the appropriate GSL functions to compute them.

---

#### Units:

Factors of  $\hbar$ ,  $c$  and  $k_B$  have been removed everywhere, so that mass, energy, and temperature all have the same units. Number and entropy densities have units of mass cubed (or energy cubed). This particle classes can be used with any system of units which is consistent with the above, i.e.  $[n] = [T]^3 = [m]^3 = [P]^{4/3}$ , etc.

---

## 1.3 Atomic nuclei

### Nuclei

Atomic nuclei, class `nucleus`, are implemented as descendants of `classical`. This class sets the value of `nucleus::inc_rest_mass` to false by default.

Nuclear mass formulas are given as children of `nuclear_mass`. The class `mnmsk_mass` provides the mass formula from [Moller95](#), and the class `ame_mass` provides the mass formula from [Audi95](#) or [Audi03](#). A simple semi-empirical mass formula is given in `semi_empirical_mass` and this can be fit to experimentally measured masses using `mass_fit`.

The class `nuclear_dist` provides an generic base class for a **collection** of several nuclei with an STL-like iterator. There are two implementations of this base class, `simple_dist` which provides a simple distribution and `full_dist` which enumerates all the nuclei for a given mass formula.

---

## 1.4 Example source code

### Example list

- [Particle example](#)

#### 1.4.1 Particle example

```
/* Example: ex_part.cpp
-----
*/

#include <cmath>
#include <o2scl/test_mgr.h>
#include <o2scl/constants.h>
```

---

```

#include <o2scl/eff_fermion.h>
#include <o2scl/rel_fermion.h>
#include <o2scl/classical.h>

using namespace std;
using namespace o2scl;
using namespace o2scl_const;

int main(void) {
    test_mgr t;
    t.set_output_level(1);

    // Create two different electrons, one using the exact method from
    // rel_fermion, and the other from the approximate scheme used in
    // eff_fermion. We work in units of inverse Fermis, so that energy
    // density is fm-4. We also use a classical particle, to compute
    eff_fermion e(o2scl_fm::mass_electron,2.0);
    rel_fermion e2(o2scl_fm::mass_electron,2.0);
    classical e3(o2scl_fm::mass_electron,2.0);

    // Compute the pressure at a density of 0.0001 fm-3 and a
    // temperature of 10 MeV. At these temperatures, the electrons are
    // non-degenerate, and Boltzmann statistics nearly applies.
    e.n=0.0001;
    e.calc_density(10.0/o2scl_const::hc_mev_fm);
    e2.n=0.0001;
    e2.calc_density(10.0/o2scl_const::hc_mev_fm);
    e3.n=0.0001;
    e3.calc_density(10.0/o2scl_const::hc_mev_fm);

    cout << e.pr << " " << e2.pr << " " << e3.pr << " "
         << e.n*10.0/hc_mev_fm << endl;

    // Compute the pressure at a density of 0.1 fm-3 and a
    // temperature of 1 MeV. At these temperatures, the electrons are
    // strongly degenerate
    e.n=0.0001;
    e.calc_density(10.0/o2scl_const::hc_mev_fm);
    e2.n=0.0001;
    e2.calc_density(10.0/o2scl_const::hc_mev_fm);
    cout << e.pr << " " << e2.pr << endl;

    // Now add the contribution to the pressure from positrons using the
    // implmentation of part::pair_density()
    e.n=0.0001;
    e.pair_density(10.0/o2scl_const::hc_mev_fm);
    e2.n=0.0001;
    e2.pair_density(10.0/o2scl_const::hc_mev_fm);
    cout << e.pr << " " << e2.pr << endl;

    t.report();
    return 0;
}
// End of example

```

---

## 1.5 Bibliography

Some of the references which contain links should direct you to the work referred to directly through [dx.doi.org](https://dx.doi.org).

Audi95: [G. Audi and A. H. Wapstra](#), Nucl. Phys. A **595** (1995) 409-480.

Audi03: [G. Audi, A. H. Wapstra and C. Thibault](#), Nucl. Phys. A **729** (2003) 337.

Eggleton73: P.P. Eggleton, J. Faulkner, and B.P. Flannery, Astron. and Astrophys. **23** (1973) 325.

Johns96: [Johns, P.J. Ellis, and J.M. Lattimer](#), Astrophys. J. **473**, (1996) 1020.

Moller95: [P. Moller, J.R. Nix, W.D. Myers, and W.J. Swiatecki](#), At. Data Nucl. Data Tables **59** (1995) 185.

## 2 Ideas for future development

**Class [ame\\_mass](#)** Create a caching and more intelligent search system for the **table**. The **table** is sorted by A and then N, so we could probably just copy the search routine from [mnmsk\\_mass](#), which is sorted by Z and then N.

**Class [ame\\_mass](#)** Use the atomic mass unit and other constants defined in the evaluation

**Class [eff\\_fermion](#)** Use bracketing to speed up one-dimensional **root** finding.

**Class [fermion](#)** Consider putting a parent version of `calc_e` and `calc_p`, or in [part](#) or [fermion](#) which automatically solves like `eff_fermion::calc_density()`?

**Class [nonrel\\_fermion](#)** This could be improved by performing a Chebyshev approximation to invert the density integral so that we don't need to use a solver.

**Class [nuclear\\_mass](#)** It might be useful to consider a fudge factor to ensure no problems with finite precision arithmetic when converting `double` to `int`.

**Global [nuclear\\_mass::parse\\_elstring](#)(std::string ela, int &Z, int &N, int &A)** Properly ignore dashes and other non-alphanumeric characters.

**Class [rel\\_fermion](#)** Allow the user to change the upper limit on the degenerate integration and the hard-coded value of 200 in the integrands.

**Class [rel\\_fermion](#)** It appears this doesn't compute the uncertainty in the chemical potential or density with `calc_density()`. This could be fixed.

## 3 Bug List

**Global [classical::pair\\_mu](#)(const double temper)** This function probably doesn't work (3/6/08). See end of `eff_fermion_ts.cpp`.

**Global [classical::pair\\_density](#)(const double temper)** This function probably doesn't work (3/6/08). See end of `eff_fermion_ts.cpp`.

---

## 4 Todo List

**Class `eff_boson`** Better documentation (see `eff_fermion`)

**Class `eff_boson`** Remove native error codes

**Global `eff_fermion::calc_mu(const double temper)`** Should see if the function actually works if  $(\mu - m)/T = -199$ .

**Class `eff_quark`** Add testing.

**Global `fermion::massless_pair_density(const double temper)`** Comment here about the precision of the expansions and allow the user to control how they are used if necessary.

**Class `mass_fit`** Convert to a real fit with errors and covariance, etc.

**Class `nonrel_fermion`** I think `calc_mu_zerot()` and `calc_density_zerot()` are missing the proper dependence on the degeneracy,  $g$ . (8/20/07) (I think this is fixed now, but should be tested, 8/22/07)

**Class `nonrel_fermion`** Make sure to test with non-interacting equal to true or false, and document whether or not it works with both `inc_rest_mass` equal to true or false

**Class `nuclear_mass`** Make the treatment of the electron binding energy contribution more consistent.

**Class `rel_boson`** Testing not completely finished.

**Class `simple_dist`** This takes a `nuclear_mass` pointer as input. It should probably be a reference instead? (3/18/08 - Yeah, I think it should be a reference.)

**Class `simple_dist`** Add error checking in constructors and set functions

**Class `sn_classical`** This does not work with `inc_rest_mass=true`

**Class `sn_fermion`** This needs to be corrected to calculate  $\sqrt{k^2 + m^{*2}} - m$  gracefully when  $m^* \approx m$ .

**Class `sn_fermion`** Call error handler if `inc_rest_mass` is true or update to properly treat the case when `inc_rest_mass` is true.

**Class `sn_fermion`** This class will have difficulty with extremely degenerate or extremely non-degenerate systems.

**Class `sn_fermion`** Create a more intelligent method for dealing with bad initial guesses for the chemical potential in `calc_density()`.

---

## 5 Data Structure Documentation

### 5.1 ame\_entry Struct Reference

```
#include <nuclear_mass.h>
```

#### 5.1.1 Detailed Description

Atomic mass entry structure.

Definition at line 444 of file nuclear\_mass.h.

#### Data Fields

- int [NMZ](#)  
*N-Z.*
- int [N](#)  
*Neutron number.*
- int [Z](#)  
*Proton number.*
- int [A](#)  
*Atomic number.*
- std::string [el](#)  
*Element name.*
- std::string [orig](#)  
*Data origin.*
- double [mass](#)  
*Mass excess.*
- double [dmass](#)  
*Mass excess uncertainty.*
- double [be](#)  
*Binding energy (given in the '95 data).*
- double [dbe](#)  
*Binding energy uncertainty (given in the '95 data).*
- double [beoa](#)  
*Binding energy / A (given in the '03 data).*
- double [dbeoa](#)  
*Binding energy / A uncertainty (given in the '03 data).*
- std::string [bdmode](#)  
*Beta decay mode.*
- double [bde](#)  
*Beta-decay energy.*
- double [dbde](#)  
*Beta-decay energy uncertainty.*
- int [A2](#)  
*?*
- double [amass](#)  
*Atomic mass.*
- double [damass](#)  
*Atomic mass uncertainty.*

The documentation for this struct was generated from the following file:

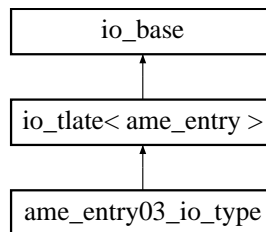
- nuclear\_mass.h



## 5.2 ame\_entry03\_io\_type Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for ame\_entry03\_io\_type::



### 5.2.1 Detailed Description

A support class for I/O of the 2003 AME data.

Definition at line 513 of file nuclear\_mass.h.

#### Public Member Functions

- **int input** (**cinput** \*co, **in\_file\_format** \*ins, [ame\\_entry](#) \*t)
- **int output** (**coutput** \*co, **out\_file\_format** \*outs, [ame\\_entry](#) \*t)
- **virtual const char \* type** ()

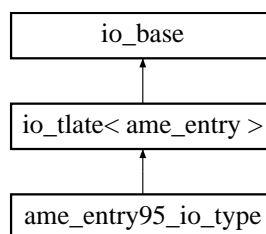
The documentation for this class was generated from the following file:

- nuclear\_mass.h

## 5.3 ame\_entry95\_io\_type Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for ame\_entry95\_io\_type::



### 5.3.1 Detailed Description

A support class for I/O of the 1995 AME data.

Definition at line 504 of file nuclear\_mass.h.

## Public Member Functions

- `int input (cinput *co, in_file_format *ins, ame_entry *t)`
- `int output (coutput *co, out_file_format *outs, ame_entry *t)`
- `virtual const char * type ()`

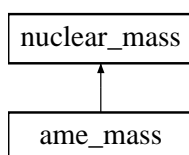
The documentation for this class was generated from the following file:

- `nuclear_mass.h`

## 5.4 ame\_mass Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for ame\_mass::



### 5.4.1 Detailed Description

Mass formula from the Atomic Mass Evaluation (2005 and 1993).

This class provides an interface to the atomic mass **table** using data from [Audi95](#) and [Audi03](#).

There are four data sets, selected by the specification of the `version` string in the constructor.

- "95rmd" - "Recommended" data from [Audi95](#) (ame95rmd.dat)
- "95exp" - "Experimental" data from [Audi95](#) (ame95exp.dat)
- "03round" - "Rounded" data from [Audi03](#) (ame03round.dat)
- "03" - Data from [Audi03](#) (default) (ame03.dat)

If any string other than these four is used, the default data is loaded. If the constructor cannot find the data file (e.g. because of a broken installation), then `ame::is_loaded()` returns false.

The 1995 data provided the binding energy stored in `ame_entry::be` and `ame_entry::dbe`, while the 2003 data provided the binding energy divided by the atomic number stored in `ame_entry::beoa` and `ame_entry::dbeoa`. When the 1995 data is used `ame_entry::beoa` and `ame_entry::dbeoa` are calculated automatically, and when the 2003 data is used `ame_entry::be` and `ame_entry::dbe` are calculated automatically.

Note that blank entries in the original **table** that correspond to columns represented by the type `double` are set to zero arbitrarily.

Note that all uncertainties are 1 sigma uncertainties.

#### Warning:

There are strict definitions of the atomic mass unit and other constants that are defined by the 1995 and 2003 atomic mass evaluations which are not used at present.

#### Idea for future

Create a caching and more intelligent search system for the **table**. The **table** is sorted by A and then N, so we could probably just copy the search routine from `mnmsk_mass`, which is sorted by Z and then N.

### Idea for future

Use the atomic mass unit and other constants defined in the evaluation

Definition at line 563 of file nuclear\_mass.h.

### Public Member Functions

- `ame_mass` (std::string version="")  
*Create a **collection** specified by version.*
- virtual bool `is_included` (int Z, int N)  
*Return false if the mass formula does not include specified **nucleus**.*
- virtual double `mass_excess` (int Z, int N)  
*Given Z and N, return the mass excess in MeV.*
- `ame_entry get_ZN` (int l\_Z, int l\_N)  
*Get element with Z=l\_Z and N=l\_N (e.g. 82,126).*
- `ame_entry get_ZA` (int l\_Z, int l\_A)  
*Get element with Z=l\_Z and A=l\_A (e.g. 82,208).*
- `ame_entry get_elA` (std::string l\_el, int l\_A)  
*Get element with name l\_el and A=l\_A (e.g. "Pb",208).*
- `ame_entry get` (std::string nucleus)  
*Get element with string (e.g. "Pb208").*
- bool `is_loaded` ()  
*Returns true if the constructor succesfully loaded the data.*

### Data Fields

- int `n`  
*The number of entries (about 3000).*
- std::string \* `short_names`  
*The short names of the columns (length 16).*
- std::string \* `col_names`  
*The long names of the columns (length 16).*
- std::string `reference`  
*The reference for the original data.*
- `ame_entry` \* `mass`  
*The array containing the mass data of length ame::n.*

### Protected Attributes

- bool `loaded`  
*True if loading the data was successful.*

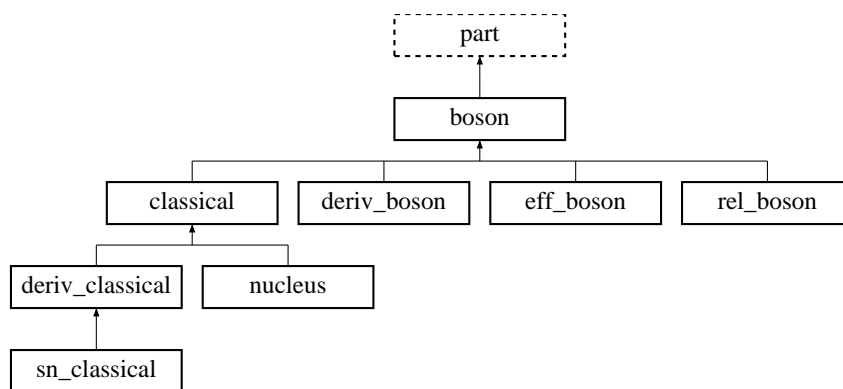
The documentation for this class was generated from the following file:

- nuclear\_mass.h

## 5.5 boson Class Reference

```
#include <boson.h>
```

Inheritance diagram for boson::



### 5.5.1 Detailed Description

Boson class.

For bosons:

- if either  $\nu$  or  $\mu$  is greater than  $m_s$ , then they are taken to be equal to  $m_s$
- All contributions from any type of condensate are ignored.

This Mathematica notebook contains the series expansions for the bosonic integrals. functions.

```

doc/o2scl/extras/boson.nb
doc/o2scl/extras/boson.pdf

```

Definition at line 66 of file boson.h.

#### Public Member Functions

- **boson** (double  $m=0.0$ , double  $g=0.0$ )  
Create a **boson** with mass  $m$  and degeneracy  $g$ .
- virtual int **calc\_mu** (const double temper)  
Calculate properties as function of chemical potential.
- virtual int **calc\_density** (const double temper)  
Calculate properties as function of density.
- virtual int **pair\_mu** (const double temper)  
Calculate properties with antiparticles as function of chemical potential.
- virtual int **pair\_density** (const double temper)  
Calculate properties with antiparticles as function of density.
- virtual int **massless\_calc\_mu** (const double temper)  
Calculate properties of massless bosons.
- virtual const char \* **type** ()  
Return string denoting type ("boson").

#### Data Fields

- double **co**  
The condensate.

## 5.5.2 Member Function Documentation

### 5.5.2.1 virtual int massless\_calc\_mu (const double *temper*) [virtual]

Calculate properties of massless bosons.

The expressions used are exact. The chemical potentials are ignored and the scalar density is set to zero

## 5.5.3 Field Documentation

### 5.5.3.1 double co

The condensate.

The condensate variable is mostly ignored by class [boson](#) and its descendants, and is provided for user storage.

Definition at line 76 of file boson.h.

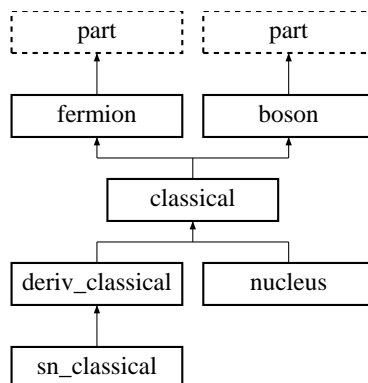
The documentation for this class was generated from the following file:

- boson.h

## 5.6 classical Class Reference

```
#include <classical.h>
```

Inheritance diagram for classical::



### 5.6.1 Detailed Description

Classical particle class.

Classical particles, because they inherit from both [fermion](#) and [boson](#), have a fermi momentum, and a condensate, but these variables are ignored.

Definition at line 48 of file classical.h.

### Public Member Functions

- [classical](#) (double [m](#)=0.0, double [g](#)=0.0)  
Create a *classical* particle with mass `m` and degeneracy `g`.
- virtual int [calc\\_mu](#) (const double *temper*)  
Calculate properties as function of chemical potential.

- virtual int [calc\\_density](#) (const double *temper*)  
*Calculate properties as function of density.*
- virtual int [pair\\_mu](#) (const double *temper*) [virtual]  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int [pair\\_density](#) (const double *temper*)  
*Calculate properties with antiparticles as function of density.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("classical").*

## 5.6.2 Member Function Documentation

### 5.6.2.1 virtual int pair\_mu (const double *temper*) [virtual]

Calculate properties with antiparticles as function of chemical potential.

#### Bug

This function probably doesn't work (3/6/08). See end of eff\_fermion\_ts.cpp.

Reimplemented from [boson](#).

Reimplemented in [sn\\_classical](#).

### 5.6.2.2 virtual int pair\_density (const double *temper*) [virtual]

Calculate properties with antiparticles as function of density.

#### Bug

This function probably doesn't work (3/6/08). See end of eff\_fermion\_ts.cpp.

Reimplemented from [boson](#).

Reimplemented in [sn\\_classical](#).

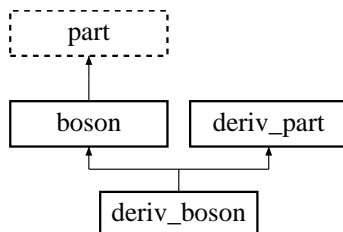
The documentation for this class was generated from the following file:

- classical.h

## 5.7 deriv\_boson Class Reference

```
#include <deriv_part.h>
```

Inheritance diagram for deriv\_boson::



### 5.7.1 Detailed Description

Boson with derivatives.

Definition at line 162 of file deriv\_part.h.

#### Public Member Functions

- **deriv\_boson** (double mass, double dof)

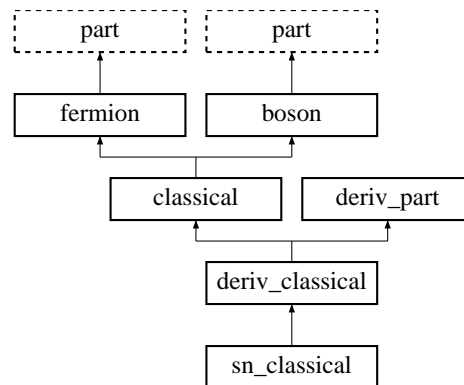
The documentation for this class was generated from the following file:

- deriv\_part.h

## 5.8 deriv\_classical Class Reference

```
#include <deriv_part.h>
```

Inheritance diagram for deriv\_classical::



### 5.8.1 Detailed Description

Classical particle with derivatives.

Definition at line 170 of file deriv\_part.h.

#### Public Member Functions

- **deriv\_classical** (double mass, double dof)

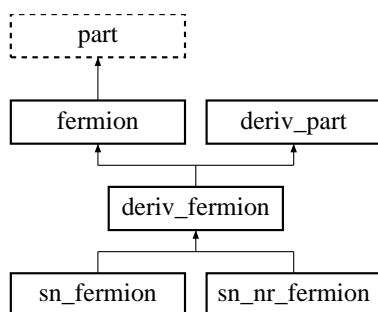
The documentation for this class was generated from the following file:

- deriv\_part.h

## 5.9 deriv\_fermion Class Reference

```
#include <deriv_part.h>
```

Inheritance diagram for deriv\_fermion::



### 5.9.1 Detailed Description

Fermion with derivatives.

Definition at line 153 of file deriv\_part.h.

#### Public Member Functions

- **deriv\_fermion** (double mass, double dof)

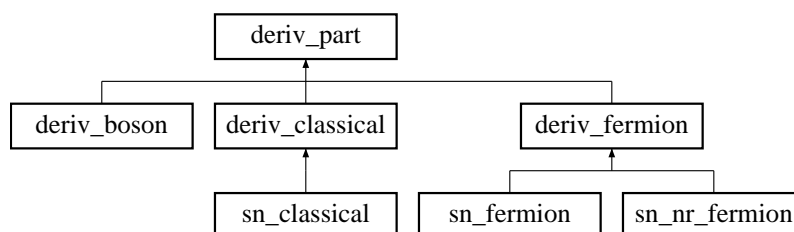
The documentation for this class was generated from the following file:

- deriv\_part.h

## 5.10 deriv\_part Class Reference

```
#include <deriv_part.h>
```

Inheritance diagram for deriv\_part::



### 5.10.1 Detailed Description

Storage for deriviatives wrt  $\mu$  and T.

The variables `dndmu`, `dndT`, and `dsdT` correspond to

$$\left(\frac{dn}{d\mu}\right)_T, \quad \left(\frac{dn}{dT}\right)_\mu, \quad \text{and} \quad \left(\frac{ds}{dT}\right)_\mu$$

respectively.

All other derivatives can be expressed simply in terms of these three.

---

**Derivatives wrt to chemical potential and temperature:**

---



There is a Maxwell relation

$$\left(\frac{ds}{d\mu}\right)_T = \left(\frac{dn}{dT}\right)_\mu$$

The pressure derivatives are trivial

$$\left(\frac{dP}{d\mu}\right)_T = n, \quad \left(\frac{dP}{dT}\right)_\mu = s$$

The energy density derivatives are related through the thermodynamic identity:

$$\begin{aligned} \left(\frac{d\varepsilon}{d\mu}\right)_T &= \mu \left(\frac{dn}{d\mu}\right)_T + T \left(\frac{ds}{d\mu}\right)_T \\ \left(\frac{d\varepsilon}{dT}\right)_\mu &= \mu \left(\frac{dn}{dT}\right)_\mu + T \left(\frac{ds}{dT}\right)_\mu \end{aligned}$$

#### Other derivatives:

Note that the derivative of the entropy with respect to the temperature above is not the specific heat,  $c_V$ . The specific heat is

$$C_V = \frac{T}{N} \left(\frac{\partial S}{\partial T}\right)_{V,N} = \frac{T}{n} \left(\frac{\partial s}{\partial T}\right)_{V,n}$$

To compute the specific heat in terms of the derivatives above, note that the descendants of [deriv\\_part](#) provide all of the thermodynamic functions in terms of  $\mu$ ,  $V$  and  $T$ , so we have

$$s = s(\mu, V, T) \quad \text{and} \quad n = n(\mu, V, T).$$

We can then construct a function

$$s = s[\mu(n, V, T), V, T]$$

and then write the required derivative directly

$$\left(\frac{\partial s}{\partial T}\right)_{n,V} = \left(\frac{\partial s}{\partial \mu}\right)_{T,V} \left(\frac{\partial \mu}{\partial T}\right)_{n,V} + \left(\frac{\partial s}{\partial T}\right)_{\mu,V}.$$

Now we use the identity

$$\left(\frac{\partial \mu}{\partial T}\right)_{n,V} = - \left(\frac{\partial n}{\partial T}\right)_{\mu,V} \left(\frac{\partial n}{\partial \mu}\right)_{T,V}^{-1},$$

and the Maxwell relation above to give

$$C_V = \frac{T}{n} \left[ \left(\frac{\partial s}{\partial T}\right)_{\mu,V} - \left(\frac{\partial n}{\partial T}\right)_{\mu,V}^2 \left(\frac{\partial n}{\partial \mu}\right)_{T,V}^{-1} \right]$$

which expresses the specific heat in terms of the three derivatives which are given.

Note that this is the specific heat per particle, and has no units. If specific heat per unit volume is required, you must multiply by the number density.

Definition at line 132 of file deriv\_part.h.

#### Data Fields

- double [dndmu](#)  
Derivative of number density with respect to chemical potential.
- double [dndT](#)  
Derivative of number density with respect to temperature.
- double [dsdT](#)

*Derivative of entropy density with respect to temperature.*

- double [dndm](#)

*Derivative of number density with respect to the effective mass.*

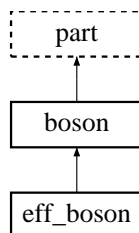
The documentation for this class was generated from the following file:

- `deriv_part.h`

## 5.11 eff\_boson Class Reference

```
#include <eff_boson.h>
```

Inheritance diagram for `eff_boson`:



### 5.11.1 Detailed Description

Boson class from fitting method.

The constructor loads the coefficients from the file `boselat3` by default. If this is not successful, then `is_loaded()` will return false.

#### Todo

Better documentation (see [eff\\_fermion](#))

#### Todo

Remove native error codes

Definition at line 59 of file `eff_boson.h`.

### Public Member Functions

- [eff\\_boson](#) (double `m`=0.0, double `g`=0.0)  
*Create a `boson` with mass `m` and degeneracy `g`.*
- virtual int [calc\\_mu](#) (const double `temper`)  
*Calculate properties as function of chemical potential.*
- virtual int [calc\\_density](#) (const double `temper`)  
*Calculate properties as function of density.*
- virtual int [pair\\_mu](#) (const double `temper`)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int [pair\\_density](#) (const double `temper`)  
*Calculate properties with antiparticles as function of density.*
- int [set\\_psi\\_root](#) (`root`< void \*, `funct`< void \* > > &`rp`)  
*Set the solver for use in calculating  $\psi$ .*
- int [set\\_density\\_mroot](#) (`mroot`< void \*, `mm_funct`< void \* > > &`rp`)  
*Set the solver for use in calculating the chemical potential from the density.*

- int [set\\_meth2\\_root](#) ([root](#)< void \*, [funct](#)< void \* > > &rp)  
*Set the solver for use in calculating the chemical potential from the density (meth2=true).*
- virtual const char \* [type](#) ()  
*Return string denoting type ("boson").*

### Static Public Member Functions

- static int [loadcoeff](#) (std::string bfile)  
*Load coefficients for finite-temperature approximation.*

### Data Fields

- [gsl\\_mroot\\_hybrids](#)< void \*, [mm\\_funct](#)< void \* > > [def\\_density\\_mroot](#)  
*The default solver for [calc\\_density\(\)](#) and [pair\\_density\(\)](#).*
- [cern\\_mroot\\_root](#)< void \*, [funct](#)< void \* > > [def\\_psi\\_root](#)  
*The default solver for  $\psi$ .*
- [cern\\_mroot\\_root](#)< void \*, [funct](#)< void \* > > [def\\_meth2\\_root](#)  
*The default solver for [calc\\_density\(\)](#) and [pair\\_density\(\)](#).*

### Protected Member Functions

- int [solve\\_fun](#) (double x, double &y, void \*&pa)  
*The function which solves for  $h$  from  $\psi$ .*
- int [density\\_fun](#) (size\_t nv, const [ovector\\_view](#) &x, [ovector\\_view](#) &y, void \*&pa)  
*Fix density for [calc\\_density\(\)](#).*
- int [pair\\_density\\_fun](#) (size\_t nv, const [ovector\\_view](#) &x, [ovector\\_view](#) &y, void \*&pa)  
*Fix density for [pair\\_density\(\)](#).*

### Protected Attributes

- [mroot](#)< void \*, [mm\\_funct](#)< void \* > > \* [density\\_mroot](#)  
*The solver for [calc\\_density\(\)](#).*
- [root](#)< void \*, [funct](#)< void \* > > \* [psi\\_root](#)  
*The solver to compute  $h$  from  $\psi$ .*
- [root](#)< void \*, [funct](#)< void \* > > \* [meth2\\_root](#)  
*The solver for [calc\\_density\(\)](#).*

### Static Protected Attributes

- static double \*\* [Pmnb](#)  
*The coefficients.*
- static int [sizem](#)  
*The number of coefficient rows.*
- static int [sizen](#)  
*The number of coefficient columns.*
- static double [parma](#)  
*The parameter,  $a$ .*
- static double [fix\\_density](#)  
*Temporary storage.*
- static double [stat\\_temper](#)  
*Temporary storage.*
- static bool [loaded](#)  
*True if coefficients have been loaded.*

## 5.11.2 Member Function Documentation

### 5.11.2.1 static int loadcoeff (std::string *bfile*) [static]

Load coefficients for finite-temperature approximation.

Presently acceptable values of *fn* are: *boselat3* from Lattimer's notes *bosejel21*, *bosejel22*, *bosejel34*, and *bosejel34cons* from [Johns96](#).

#### *boselat3*

```
double a 1.038
double[][] Pmn 4 4
1.68123 5.17553 5.66067 2.16447
6.72492 20.70212 22.64268 8.65788
8.51035 27.1555 30.5655 11.8288
3.47086 11.7253 13.8574 5.53865
```

#### *bosejel21*

```
double a 0.978
double[][] Pmn 3 2
1.63146 2.11571
4.89438 6.34713
3.31275 5.15372
```

#### *bosejel22*

```
double a 0.914
double[][] Pmn 3 3
1.68131 3.47558 2.16582
5.04393 10.42674 6.49746
3.25053 7.82859 5.19126
```

#### *bosejel34*

```
double a 1.029
double[][] Pmn 4 5
1.68134 6.85070 10.8537 7.81843 2.16461
6.72536 27.4028 43.4148 31.2737 8.65844
8.49651 35.6058 57.7134 42.3593 11.8199
3.45614 15.1152 25.5254 19.2745 5.51757
```

#### *bosejel34cons*

```
double a 1.040
double[][] Pmn 4 5
1.68130 6.85060 10.8539 7.81762 2.16465
6.72520 27.40240 43.4156 31.27048 8.65860
8.51373 35.6576 57.7975 42.4049 11.8321
3.47433 15.1995 25.6536 19.3811 5.54423
```

The documentation for this class was generated from the following file:

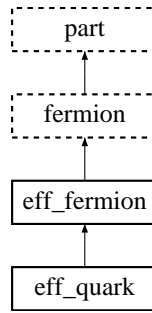
- *eff\_boson.h*

## 5.12 eff\_fermion Class Reference

```
#include <eff_fermion.h>
```

---

Inheritance diagram for eff\_fermion::



### 5.12.1 Detailed Description

Fermion class from fitting method.

Based on the fitting method of [Johns96](#) which is an update of the method from [Eggleton73](#) . This method is approximate, but very fast. For a more accurate (but slower) method, use [rel\\_fermion](#).

Given the chemical potential and the temperature the functions [calc\\_mu\(\)](#) and [pair\\_mu\(\)](#) work by solving the equation

$$\psi = 2\sqrt{1 + f/a} + \log \left( \frac{\sqrt{1 + f/a} - 1}{\sqrt{1 + f/a} + 1} \right)$$

for  $f$  given  $\psi = (\mu - m)/T$ . If  $f/a < 10^{-10}$ , then the alternative expression

$$\psi = 2(1 + f/(2a)) + \log \left[ \frac{f/(2a)}{(1 + f/(2a))} \right]$$

is used. The pressure, energy density, and entropy, are determined as polynomials in  $f$  with a set of precomputed coefficients as done in [Johns96](#) .

If  $\psi$  is too small (less than about -200), the above procedure fails. To handle this, this class uses the [classical](#) result if  $\psi < \text{min\_psi}$ , where [min\\_psi](#) defaults to -200.

When the density and temperature is given instead ([calc\\_density\(\)](#) and [pair\\_density\(\)](#)), then there are two ways to proceed.

- We can use the density to solve for  $f$
- We can use the density to solve for the chemical potential

Because the density is a complicated polynomial in  $f$ , the former procedure does not work very well even though it might be less time consuming. The density is solved for the effective chemical potential instead. The initial guess is just taken from the present value of [part::nu](#).

The constructor uses the coefficients from the file `fermilat3` by default.

It is important that if the **inte** and **root** objects are not thread safe, different instances should not use the same instance of a **inte** or **root** object simultaneously.

#### Idea for future

Use bracketing to speed up one-dimensional **root** finding.

Definition at line 94 of file `eff_fermion.h`.

## Load coefficients for finite-temperature approximation

ctype Should be one of the constants below: cf\_fermilat3, cf\_fermijel2, cf\_fermijel3, or cf\_fermijel3cons.

### Todo

Document these coefficients better

- static const int **cf\_fermilat3** = 1
- static const int **cf\_fermijel2** = 2
- static const int **cf\_fermijel3** = 3
- static const int **cf\_fermijel3cons** = 4
- static int **load\_coefficients** (int ix)

## Public Member Functions

- **eff\_fermion** (double mass=0.0, double dof=0.0)  
*Create a [fermion](#) with mass `mass` and degeneracy `dof`.*
- virtual int **calc\_mu** (const double temper)  
*Calculate thermodynamic properties as function of chemical potential.*
- virtual int **calc\_density** (const double temper)  
*Calculate thermodynamic properties as function of density.*
- virtual int **pair\_mu** (const double temper)  
*Calculate thermodynamic properties with antiparticles as function of chemical potential.*
- virtual int **pair\_density** (const double temper)  
*Calculate thermodynamic properties with antiparticles as function of density.*
- int **set\_psi\_root** (**root**< void \*, **funct**< void \* > > &rp)  
*Set the solver for use in calculating  $\psi$ .*
- int **set\_density\_root** (**root**< void \*, **funct**< void \* > > &rp)  
*Set the solver for use in calculating the chemical potential from the density with `meth2=true`.*
- virtual const char \* **type** ()  
*Return string denoting type ("eff\_fermion").*

## Data Fields

- double **tlimit**  
*If the temperature is less than `tlimit` then the zero-temperature functions are used (default 0).*
- **cern\_mroot\_root**< void \*, **funct**< void \* > > **def\_psi\_root**  
*The default solver for  $\psi$ .*
- **cern\_mroot\_root**< void \*, **funct**< void \* > > **def\_density\_root**  
*The default solver for [calc\\_density\(\)](#) and [pair\\_density\(\)](#).*
- double **min\_psi**  
*The minimum value of  $\psi$  (default -200).*

## Protected Member Functions

- int **solve\_fun** (double x, double &y, void \*&pa)  
*The function which solves for  $f$  from  $\psi$ .*
- int **density\_fun** (double x, double &y, void \*&pa)  
*Fix density for [calc\\_density\(\)](#).*
- int **pair\_density\_fun** (double x, double &y, void \*&pa)  
*Fix density for [pair\\_density\(\)](#).*

## Protected Attributes

- **root**< void \*, **funct**< void \* > > \* [psi\\_root](#)  
*The solver for  $\psi$ .*
- **root**< void \*, **funct**< void \* > > \* [density\\_root](#)  
*The other solver for [calc\\_density\(\)](#).*

## Static Protected Attributes

- static double \*\* [Pmnf](#)  
*The matrix of coefficients.*
- static double [parma](#)  
*The parameter  $a$ .*
- static int [sizem](#)  
*The array row size.*
- static int [sizen](#)  
*The array column size.*

## 5.12.2 Member Function Documentation

### 5.12.2.1 virtual int calc\_mu (const double temper) [virtual]

Calculate thermodynamic properties as function of chemical potential.

If the quantity  $(\mu - m)/T$  (or  $(\nu - m^*)/T$  in the case of interacting particles) is less than -200, then this quietly sets the density, the scalar density, the energy density, the pressure and the entropy to zero and exits.

#### Todo

Should see if the function actually works if  $(\mu - m)/T = -199$ .

Reimplemented from [fermion](#).

Reimplemented in [eff\\_quark](#).

### 5.12.2.2 virtual int calc\_density (const double temper) [virtual]

Calculate thermodynamic properties as function of density.

#### Warning:

This function needs a guess for the chemical potential, and will fail if that guess is not sufficiently accurate.

Reimplemented from [fermion](#).

Reimplemented in [eff\\_quark](#).

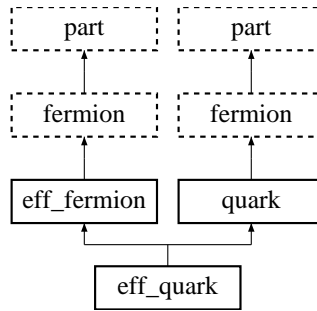
The documentation for this class was generated from the following file:

- [eff\\_fermion.h](#)

## 5.13 eff\_quark Class Reference

```
#include <eff_quark.h>
```

Inheritance diagram for `eff_quark`:



### 5.13.1 Detailed Description

Quark class from fitting method.

#### Todo

Add testing.

Definition at line 45 of file eff\_quark.h.

### Public Member Functions

- **eff\_quark** (double **m**=0.0, double **g**=0.0)  
*Create a **quark** with mass **m** and degeneracy **g**.*
- virtual int **calc\_mu** (const double **temper**)  
*Calculate thermodynamic properties as function of chemical potential.*
- virtual int **calc\_density** (const double **temper**)  
*Calculate thermodynamic properties as function of density.*
- virtual int **pair\_mu** (const double **temper**)  
*Calculate thermodynamic properties with antiparticles as function of chemical potential.*
- virtual int **pair\_density** (const double **temper**)  
*Calculate thermodynamic properties with antiparticles as function of density.*
- virtual const char \* **type** ()  
*Return string denoting type ("eff\_quark").*

### 5.13.2 Member Function Documentation

#### 5.13.2.1 virtual int calc\_mu (const double *temper*) [virtual]

Calculate thermodynamic properties as function of chemical potential.

If the quantity  $(\mu - m)/T$  (or  $(\nu - m^*)/T$  in the case of interacting particles) is less than -200, then this quietly sets the density, the scalar density, the energy density, the pressure and the entropy to zero and exits.

#### Todo

Should see if the function actually works if  $(\mu - m)/T = -199$ .

Reimplemented from [eff\\_fermion](#).

#### 5.13.2.2 virtual int calc\_density (const double *temper*) [virtual]

Calculate thermodynamic properties as function of density.



**Warning:**

This function needs a guess for the chemical potential, and will fail if that guess is not sufficiently accurate.

Reimplemented from [eff\\_fermion](#).

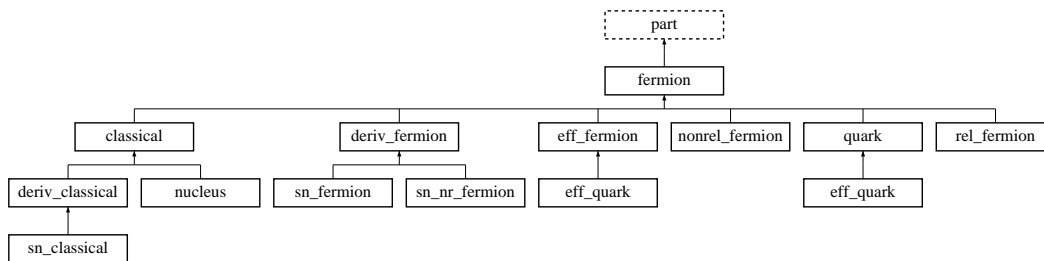
The documentation for this class was generated from the following file:

- `eff_quark.h`

**5.14 fermion Class Reference**

```
#include <fermion.h>
```

Inheritance diagram for fermion::

**5.14.1 Detailed Description**

Fermion class.

This is a base class for the computation of fermionic thermodynamics. This class includes the computations of zero-temperature (possibly massive) and massless fermions at zero or finite temperature. The more general case of fermions with both finite mass and a finite temperature are taken care of by the functions `calc_mu()`, `calc_density()`, `pair_mu()`, and `pair_density()`. These functions are not implemented in this base class (see `eff_fermion`, `rel_fermion`, `nonrel_fermion`, `sn_fermion`, and `sn_nr_fermion`).

The function `massless_calc_density()` uses a `root` object to solve for the chemical potential as a function of the density. The default is an object of type `cern_mroot_root`. The function `massless_pair_density()` does not need to use the `root` object because of the simplification afforded by the inclusion of antiparticles.

**Idea for future**

Consider putting a parent version of `calc_e` and `calc_p`, or in `part` or `fermion` which automatically solves like `eff_fermion::calc_density()`?

This Mathematica notebook contains the derivations of related series expansions and some algebra for the `massless_pair()` functions.

```
doc/o2scl/extras/fermion.nb
doc/o2scl/extras/fermion.pdf
```

Definition at line 86 of file `fermion.h`.

**Public Member Functions**

- `fermion` (double mass=0, double dof=0)  
Create a `fermion` with mass `mass` and degeneracy `dof`.

- virtual int `calc_mu` (const double temper)  
*Calculate properties as function of chemical potential.*
- virtual int `calc_density` (const double temper)  
*Calculate properties as function of density.*
- virtual int `pair_mu` (const double temper)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int `pair_density` (const double temper)  
*Calculate properties with antiparticles as function of density.*
- int `set_massless_root` (**root**< void \*, **funct**< void \* > > &rp)  
*Set the solver for use in massless\_root\_density().*
- double `deg_specific_heat` (double T)  
*Degenerate expansion for specific heat.*
- virtual const char \* `type` ()  
*Return string denoting type ("fermion").*

### Zero-temperature fermions

- int `kffromden` ()  
*Calculate the Fermi momentum from the density.*
- int `sden` ()  
*Scalar number density at T=0 from `kf` and `ms`.*
- int `eden` ()  
*Energy density at T=0 from `kf` and `ms`.*
- int `pres` ()  
*Pressure at T=0 from `kf` and `ms`.*
- virtual int `calc_mu_zerot` ()  
*Zero temperature fermions from `nu` and `ms`.*
- virtual int `calc_density_zerot` ()  
*Zero temperature fermions from `n` and `ms`.*

### Massless fermions

- virtual int `massless_calc_mu` (const double temper)  
*Finite temperature massless fermions.*
- virtual int `massless_calc_density` (const double temper)  
*Finite temperature massless fermions.*
- int `massless_pair_mu` (const double temper)  
*Finite temperature massless fermions and antifermions.*
- int `massless_pair_density` (const double temper)  
*Finite temperature massless fermions and antifermions.*

### Data Fields

- double `kf`  
*Fermi momentum.*
- double `del`  
*Gap.*
- **cern\_mroot\_root**< void \*, **funct**< void \* > > `def_massless_root`  
*The default solver for `massless_calc_mu`().*

## 5.14.2 Member Function Documentation

### 5.14.2.1 int kffromden ()

Calculate the Fermi momentum from the density.

Uses the relation  $k_F = (6\pi^2 n/g)^{1/3}$

---

**5.14.2.2 int sden ()**

Scalar number density at T=0 from [kf](#) and [ms](#).

Calculates the integral

$$n_s = \frac{g}{2\pi^2} \int_0^{k_F} k^2 \frac{m^*}{\sqrt{k^2 + m^{*2}}} dk$$

**5.14.2.3 int eden ()**

Energy density at T=0 from [kf](#) and [ms](#).

Calculates the integral

$$\varepsilon = \frac{g}{2\pi^2} \int_0^{k_F} k^2 \sqrt{k^2 + m^{*2}} dk$$

**5.14.2.4 int pres ()**

Pressure at T=0 from [kf](#) and [ms](#).

Calculates the integral

$$P = \frac{g}{6\pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{k^2 + m^{*2}}} dk$$

**5.14.2.5 virtual int calc\_mu\_zerot () [virtual]**

Zero temperature fermions from nu and ms.

This function always returns `gsl_success`.

Reimplemented in [nonrel\\_fermion](#).

**5.14.2.6 virtual int calc\_density\_zerot () [virtual]**

Zero temperature fermions from n and ms.

This function always returns `gsl_success`.

Reimplemented in [nonrel\\_fermion](#).

**5.14.2.7 int massless\_pair\_density (const double temper)**

Finite temperature massless fermions and antifermions.

In the cases  $n^3 \gg T$  and  $T \gg n^3$ , expansions are used instead of the exact formulas to avoid loss of precision.

**Todo**

Comment here about the precision of the expansions and allow the user to control how they are used if necessary.

**5.14.2.8 double deg\_specific\_heat (double T) [inline]**

Degenerate expansion for specific heat.

This is a temporary location and is also unchecked.

Definition at line 212 of file fermion.h.

---

### 5.14.3 Field Documentation

#### 5.14.3.1 cern\_mroot\_root<void \*,funct<void \*> > def\_massless\_root

The default solver for [massless\\_calc\\_mu\(\)](#).

We default to **cern\_mroot\_root** here since we don't have a bracket or a derivative.

Definition at line 232 of file fermion.h.

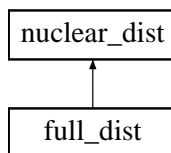
The documentation for this class was generated from the following file:

- fermion.h

## 5.15 full\_dist Class Reference

```
#include <nuclear_dist.h>
```

Inheritance diagram for full\_dist::



### 5.15.1 Detailed Description

Full distribution including all nuclei from a discrete mass formula.

For example, to create a **collection** of all nuclei from the most recent (2003) Atomic Mass Evaluation, and then output all the nuclei in the **collection**

```
ame_mass ame;
full_dist fd(&ame);
for(nuclear_dist::iterator ndi=fd.begin();ndi!=fd.end();ndi++) {
    cout << ndi->Z << " " << ndi->A << " " << ndi->m << endl;
}
```

Definition at line 264 of file nuclear\_dist.h.

### Public Member Functions

- **full\_dist** ([nuclear\\_mass](#) \*nm, int maxA=400, bool include\_neutron=false)  
*Create a distribution including all nuclei with atomic numbers less than maxA from the mass formula nm.*
- int **set\_dist** ([nuclear\\_mass](#) \*nm, int maxA=400, bool include\_neutron=false)  
*Set the distribution to all nuclei with atomic numbers less than maxA from the mass formula nm.*
- virtual [iterator](#) **begin** ()  
*The beginning of the distribution.*
- virtual [iterator](#) **end** ()  
*The end of the distribution.*
- virtual [size\\_t](#) **size** ()  
*The number of nuclei in the distribution.*

## 5.15.2 Member Function Documentation

### 5.15.2.1 int set\_dist (nuclear\_mass \* nm, int maxA = 400, bool include\_neutron = false)

Set the distribution to all nuclei with atomic numbers less than maxA from the mass formula nm.

The information for the previous distribution is cleared before a new distribution is set.

The documentation for this class was generated from the following file:

- nuclear\_dist.h

## 5.16 mass\_fit Class Reference

```
#include <mass_fit.h>
```

### 5.16.1 Detailed Description

Fit a nuclear mass formula.

#### Todo

Convert to a real fit with errors and covariance, etc.

Definition at line 43 of file mass\_fit.h.

### Public Member Functions

- virtual int [fit](#) ([nuclear\\_mass\\_fit](#) &n, double &res)  
*Fit the nuclear mass formula.*
- virtual int [eval](#) ([nuclear\\_mass](#) &n, double &res)  
*Evaluate quality without fitting.*
- int [set\\_mmin](#) ([multi\\_min](#)< void \*, [multi\\_funct](#)< void \* > > &umm)  
*Change the minimizer for use in the fit.*
- int [set\\_dist](#) ([nuclear\\_dist](#) &und)  
*Set the distribution of nuclei to fit.*
- int [set\\_masses](#) ([nuclear\\_mass](#) &uexp)  
*Set the experimental values to fit to.*

### Data Fields

- bool [even\\_even](#)  
*If true, then only fit doubly-even nuclei (default false).*
- int [minZ](#)  
*Minimum proton number to fit (default 8).*
- int [minN](#)  
*Minimum neutron number to fit (default 8).*
- [gsl\\_mmin\\_simp](#)< void \*, [multi\\_funct](#)< void \* > > [def\\_mmin](#)  
*The default minimizer.*
- [full\\_dist](#) [def\\_dist](#)  
*The default distribution of nuclei to fit (defaults to all nuclei in [def\\_exp\\_mass](#)).*
- [ame\\_mass](#) [def\\_exp\\_mass](#)  
*The default experimental nuclear mass object.*

## 5.16.2 Field Documentation

### 5.16.2.1 gsl\_mmin\_simp<void \*,multi\_func<void \*> > def\_mmin

The default minimizer.

The value of def\_mmin::ntrial is automatically multiplied by 10 in the constructor because the minimization frequently requires more trials than the default.

Definition at line 74 of file mass\_fit.h.

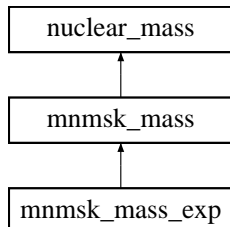
The documentation for this class was generated from the following file:

- mass\_fit.h

## 5.17 mnmsk\_mass Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for mnmsk\_mass::



### 5.17.1 Detailed Description

Mass formula from Moller, Nix, Myers and Swiatecki.

The data containing an object of type moller\_mass\_entry for 8979 nuclei is automatically loaded by the constructor. If the file (nnmc/moller.dat) is not found, then [is\\_loaded\(\)](#) will return false and all calls to [get\\_ZN\(\)](#) will return an object with  $N=Z=0$ .

There are several entries in the original **table** which are blank because they are in some way not known, measured, or computable. To distinguish these values from zero, blank entries have been replaced by the number  $1.0 \times 10^9$ . For convenience, this value is returned by [blank\(\)](#).

Definition at line 761 of file nuclear\_mass.h.

### Public Member Functions

- virtual bool [is\\_included](#) (int Z, int N)  
*Return false if the mass formula does not include specified **nucleus**.*
- virtual double [mass\\_excess](#) (int Z, int N)  
*Given Z and N, return the mass excess in MeV.*
- [mnmsk\\_mass\\_entry](#) [get\\_ZN](#) (int l\_Z, int l\_N)  
*Get the entry for the specified proton and neutron number.*
- bool [is\\_loaded](#) ()  
*Verify that the constructor properly loaded the **table**.*
- double [blank](#) ()  
*The value which corresponds to a blank entry.*
- double [neither](#) ()  
*Neither beta+ or beta- is possible.*
- double [beta\\_stable](#) ()

*The value which corresponds to a blank entry.*

- double [beta\\_plus\\_and\\_minus](#) ()  
*Both beta+ and beta- are possible.*
- double [greater\\_100](#) ()  
*The value is greater than 100.*
- double [very\\_large](#) ()  
*The value is greater than  $10^{20}$ .*

### Data Fields

- int [n](#)  
*The number of **table** entries.*
- [mnmsk\\_mass\\_entry](#) \* [mass](#)  
*The array containing the **table**.*

### Protected Attributes

- bool [loaded](#)  
*True if the **table** was successfully loaded.*
- int [last](#)  
*The last **table** index for caching.*

## 5.17.2 Member Function Documentation

### 5.17.2.1 mnmsk\_mass\_entry get\_ZN (int *l\_Z*, int *l\_N*)

Get the entry for the specified proton and neutron number.

This method searches the **table** using a cached binary search algorithm. It is assumed that the **table** is sorted first by proton number and then by neutron number.

The documentation for this class was generated from the following file:

- [nuclear\\_mass.h](#)

## 5.18 mnmsk\_mass\_entry Struct Reference

```
#include <nuclear_mass.h>
```

### 5.18.1 Detailed Description

Mass formula entry structure for Moller, et al.

Definition at line 627 of file [nuclear\\_mass.h](#).

### Data Fields

- int [N](#)  
*Neutron number.*
- int [Z](#)  
*Proton number.*
- int [A](#)  
*Atomic number.*
- double [Emic](#)

*The ground-state microscopic energy.*

- double [Mth](#)

*The theoretical mass excess (in MeV).*

- double [Mexp](#)

*The experimental mass excess (in MeV).*

- double [sigmaexp](#)

*Experimental mass excess error.*

- double [EmicFL](#)

*The ground-state microscopic energy in the FRLDM.*

- double [MthFL](#)

*The theoretical mass excess in the FRLDM.*

- std::string [spinp](#)

*Spin and parity of odd proton.*

- std::string [spinn](#)

*Spin and parity of odd neutron.*

- double [gapp](#)

*Lipkin-Nogami proton gap.*

- double [gapn](#)

*Lipkin-Nogami neutron gap.*

- double [be](#)

*Total binding energy.*

- double [S1n](#)

*One neutron separation energy.*

- double [S2n](#)

*Two neutron separation energy.*

- double [PA](#)

*Percentage of daughters generated in beta decay after beta-delayed neutron emission.*

- double [PAm1](#)

*Desc.*

- double [PAm2](#)

*Desc.*

- double [Qbeta](#)

*Energy released in beta-decay.*

- double [Tbeta](#)

*Half-life w.r.t. GT beta-decay.*

- double [S1p](#)

*One proton separation energy.*

- double [S2p](#)

*Two proton separation energy.*

- double [Qalpha](#)

*Energy released in alpha-decay.*

- double [Talpha](#)

*Half-life w.r.t. alpha-decay.*

#### Ground state deformations (perturbed-spheroid parameterization)

- double [eps2](#)

*Quadrupole.*

- double [eps3](#)

*Octupole.*

- double [eps4](#)

*Hexadecapole.*

- double [eps6](#)

*Hexacontatetrapole.*

- double [eps6sym](#)

*Hexacontatetrapole without mass asymmetry.*

#### Ground state deformations in the spherical-harmonics expansion



- double [beta2](#)  
*Quadrupole.*
- double [beta3](#)  
*Octupole.*
- double [beta4](#)  
*Hexadecapole.*
- double [beta6](#)  
*Hexacontatetrapole.*

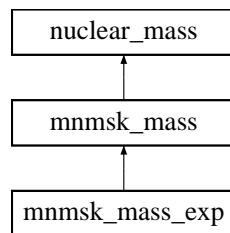
The documentation for this struct was generated from the following file:

- `nuclear_mass.h`

## 5.19 mnmsk\_mass\_exp Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for `mnmsk_mass_exp`:



### 5.19.1 Detailed Description

The experimental values from Moller, Nix, Myers and Swiatecki.

Definition at line 831 of file `nuclear_mass.h`.

#### Public Member Functions

- virtual bool [is\\_included](#) (int Z, int N)  
*Return false if the mass formula does not include specified [nucleus](#).*
- virtual double [mass\\_excess](#) (int Z, int N)  
*Given Z and N, return the mass excess in MeV.*

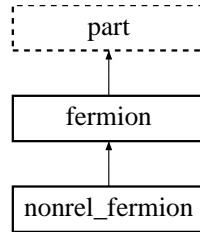
The documentation for this class was generated from the following file:

- `nuclear_mass.h`

## 5.20 nonrel\_fermion Class Reference

```
#include <nonrel_fermion.h>
```

Inheritance diagram for `nonrel_fermion`:



### 5.20.1 Detailed Description

Nonrelativistic [fermion](#) class.

The rest mass energy density is given by  $n*m$  not  $n*ms$ . Note that the effective mass here is the Landau mass, not the Dirac mass.

Pressure is computed with

$$P = 2\varepsilon/3$$

and entropy density with

$$s = \frac{5\varepsilon}{3T} - \frac{n\mu}{T}$$

These relations can be verified with an integration by parts. See, e.g. Callen's "Thermodynamics and an introduction to thermostatics", 2nd edition, pg. 403 or Landau and Lifshitz, Stat. Phys. 3rd edition, [part](#) 1, pg. 164.

Note that the energy density integral can be rescaled:

$$\varepsilon = \frac{T^{5/2}(2m^*)^{3/2}}{\pi^2} \int_0^\infty du \frac{u^{3/2}}{1 + \exp(u - y)}$$

where  $u = k^2/2/m^*/T$  and  $y = \mu/T$ .

The functions [fermion::pair\\_density\(\)](#) and [pair\\_mu\(\)](#) have not been implemented.

#### Todo

I think [calc\\_mu\\_zerot\(\)](#) and [calc\\_density\\_zerot\(\)](#) are missing the proper dependence on the degeneracy,  $g$ . (8/20/07) (I think this is fixed now, but should be tested, 8/22/07)

#### Todo

Make sure to test with non-interacting equal to true or false, and document whether or not it works with both `inc_rest_mass` equal to true or false

#### Idea for future

This could be improved by performing a Chebyshev approximation to invert the density integral so that we don't need to use a solver.

Definition at line 83 of file `nonrel_fermion.h`.

### Public Member Functions

- [nonrel\\_fermion](#) (double `m`=0.0, double `g`=0.0)  
Create a nonrelativistic [fermion](#) with mass '`m`' and degeneracy '`g`'.
- virtual int [calc\\_mu\\_zerot](#) ()  
Zero temperature fermions.
- virtual int [calc\\_density\\_zerot](#) ()  
Zero temperature fermions.

- virtual int [calc\\_mu](#) (const double temper)  
*Calculate properties as function of chemical potential.*
- virtual int [calc\\_density](#) (const double temper)  
*Calculate properties as function of density.*
- virtual int [nu\\_from\\_n](#) (const double temper)  
*Calculate effective chemical potential from density.*
- int [set\\_density\\_root](#) (**root**< double, **funct**< double > > &rp)  
*Set the solver for use in calculating the chemical potential from the density.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("nonrel\_fermion").*

## Data Fields

- **cern\_mroot\_root**< double, **funct**< double > > [def\\_density\\_root](#)  
*The default solver for [calc\\_density\(\)](#).*

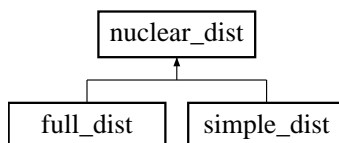
The documentation for this class was generated from the following file:

- nonrel\_fermion.h

## 5.21 nuclear\_dist Class Reference

```
#include <nuclear_dist.h>
```

Inheritance diagram for nuclear\_dist::



### 5.21.1 Detailed Description

A distribution of nuclei.

The virtual base class for a **collection** of objects of type [nucleus](#) . See [simple\\_dist](#) and [full\\_dist](#) for implementations of this base class.

Definition at line 41 of file nuclear\_dist.h.

## Public Member Functions

- virtual [iterator begin](#) ()=0  
*The beginning of the distribution.*
- virtual [iterator end](#) ()=0  
*The end of the distribution.*
- virtual size\_t [size](#) ()=0  
*The number of nuclei in the distribution.*

## Data Structures

- class [iterator](#)  
*An [iterator](#) for the nuclear distribution.*

The documentation for this class was generated from the following file:

- nuclear\_dist.h

## 5.22 nuclear\_dist::iterator Class Reference

```
#include <nuclear_dist.h>
```

### 5.22.1 Detailed Description

An [iterator](#) for the nuclear distribution.

The standard usage of this [iterator](#) is something of the form:

```
mnmsk_mass mth;
simple_dist sd(5,6,10,12,&mth);
for(nuclear_dist::iterator ndi=sd.begin();ndi!=sd.end();ndi++) {
    // do something here for each nucleus
}
```

which would create a list consisting of three isotopes (A=10, 11, and 12) of boron and three isotopes carbon for a total of six nuclei.

Definition at line 70 of file nuclear\_dist.h.

### Public Member Functions

- [iterator](#) ([nuclear\\_dist](#) \*ndpp, [nucleus](#) \*npp)  
*Create an [iterator](#) from the given distribution using the [nucleus](#) specified in npp.*
- [iterator operator++](#) ()  
*Proceed to the next [nucleus](#).*
- [iterator operator++](#) (int unused)  
*Proceed to the next [nucleus](#).*
- [nucleus](#) \* [operator](#) → () const  
*Dereference the [iterator](#).*

### Protected Attributes

- [nucleus](#) \* np
- [nuclear\\_dist](#) \* ndp  
*A pointer to the distribution.*

### Friends

- int [operator==](#) (const [nuclear\\_dist::iterator](#) &i1, const [nuclear\\_dist::iterator](#) &i2)  
*Compare two nuclei.*
- int [operator!=](#) (const [nuclear\\_dist::iterator](#) &i1, const [nuclear\\_dist::iterator](#) &i2)  
*Compare two nuclei.*

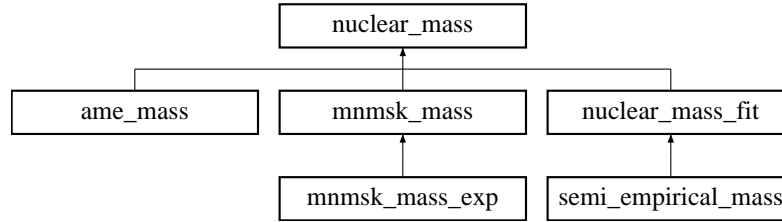
The documentation for this class was generated from the following file:

- nuclear\_dist.h

## 5.23 nuclear\_mass Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for nuclear\_mass::



### 5.23.1 Detailed Description

Nuclear mass formula base.

This base class provides some default functionality for the nuclear mass formulas. For typical usage, use [ame\\_mass](#), [mnmsk\\_mass](#), [mnmsk\\_mass\\_exp](#), or [semi\\_empirical\\_mass](#).

Elements 112-118 are named "Uub", "Uut", "Uuq", "Uup", "Uuh", "Uus", and "Uuo", respectively.

The binding energy is defined by

$$BE = Zm_H + Nm_n - m_{\text{nuclide}}$$

where  $m_{\text{nuclide}}$  is the mass of the [nucleus](#) including the mass of the electrons. The mass excess is defined by

$$m_{\text{excess}} = m_{\text{nuclide}} - Am_u$$

For example, for  $\text{U}^{238}$ , the binding energy is 1801.695 MeV, the mass excess is 47.30366 MeV, and  $m_{\text{nuclide}}$  is 221742.9 MeV. This is consistent with the above, as  $m_H$  is 938.7830 MeV,  $m_n$  is 939.5650 MeV, and  $m_u$  is 931.494 MeV.

Some mass formulas are undefined for sufficiently exotic nuclei. You can use the function [is\\_included\(\)](#) to find if a particular [nucleus](#) is included or not.

This class is designed so that a child only needs to reimplement either [mass\\_excess\(\)](#) (if `form_type==disc_type`) or [mass\\_excess\\_d\(\)](#) (if `form_type==cont_type`), and the remaining functions should work then automatically.

#### Warning:

The spin degeneracy is not handled particularly intelligently. The [get\\_nucleus\(\)](#) function simply assumes 0 spin for even A and spin 1/2 for odd A.

The treatment of the electron binding energy contribution is not necessarily consistent at present.

Some common reaction Q-values and separation energies:

$Q(\beta^-) = M(A, Z) - M(A, Z + 1)$ : Beta-decay energy

$Q(2\beta^-) = M(A, Z) - M(A, Z + 2)$ : Double beta-decay energy

$Q(4\beta^-) = M(A, Z) - M(A, Z + 4)$ : Four beta-decay energy

$Q(\alpha) = M(A, Z) - M(A - 4, Z - 2) - M(\text{He}^4)$ : Alpha-decay energy

$Q(\beta - n) = M(A, Z) - M(A - 1, Z + 1) - M(n)$ : Beta-delayed neutron emission decay energy

$Q(d, \alpha) = M(A, Z) - M(A - 2, Z - 1) - M(\text{He}^4) - M(\text{H}^2)$ :  $(d, \alpha)$  reaction energy

$Q(\text{EC}) = M(A, Z) - M(A, Z - 1)$ : Electron capture decay energy

$Q(\text{ECp}) = M(A, Z) - M(A - 1, Z - 2)$ : Electron capture with delayed proton emission decay energy

$Q(n, \alpha) = M(A, Z) - M(A - 3, Z - 2) - M(\text{He}^4) + M(n)$ :  $(n, \alpha)$  reaction energy

$Q(p, \alpha) = M(A, Z) - M(A - 3, Z - 1) - M(\text{He}^4) + M(p)$ :  $(p, \alpha)$  reaction energy

$S(n) = -M(A, Z) + M(A - 1, Z) + M(n)$ : Neutron separation energy

$S(p) = -M(A, Z) + M(A - 1, Z - 1) + M(\text{H}^1)$ : Proton separation energy

$S(2n) = -M(A, Z) + M(A - 2, Z) + 2M(n)$ : Two neutron separation energy

$S(2p) = -M(A, Z) + M(A - 2, Z - 2) + 2M(\text{H}^1)$ : Two proton separation energy

#### Note:

All descendants ought to set `form_type` to indicate whether the mass formula is discrete or continuous

#### Todo

Make the treatment of the electron binding energy contribution more consistent.

#### Idea for future

It might be useful to consider a fudge factor to ensure no problems with finite precision arithmetic when converting `double` to `int`.

Definition at line 127 of file `nuclear_mass.h`.

#### Indicate whether or not the mass formula is

discrete or continuous

- static const int **cont\_type** = 1
- static const int **disc\_type** = 2
- int **form\_type**

#### Public Member Functions

- virtual bool **is\_included** (int Z, int N)  
*Return false if the mass formula does not include specified [nucleus](#).*
- int **get\_nucleus** (int Z, int N, [nucleus](#) &n)  
*Fill n with the information from [nucleus](#) with the given neutron and proton number.*
- virtual double **mass\_excess** (int Z, int N)  
*Given Z and N, return the mass excess in MeV.*
- virtual double **mass\_excess\_d** (double Z, double N)  
*Given Z and N, return the mass excess in MeV.*
- virtual double **binding\_energy** (int Z, int N)  
*Return the binding energy in MeV.*
- virtual double **binding\_energy\_d** (double Z, double N)  
*Return the binding energy in MeV.*
- virtual double **total\_mass** (int Z, int N)  
*Return the total mass of the [nucleus](#) (without the electrons) in MeV.*
- virtual double **total\_mass\_d** (double Z, double N)  
*Return the total mass of the [nucleus](#) (without the electrons) in MeV.*
- int **eltoZ** (std::string el)  
*Return Z given the element name.*
- std::string **Ztoel** (size\_t Z)  
*Return the element name given Z.*
- int **parse\_elstring** (std::string ela, int &Z, int &N, int &A)  
*Parse a string of the form "Pb208".*

## Protected Types

- typedef std::map< std::string, int, [string\\_less\\_than](#) >::iterator [table\\_it](#)  
*A convenient typedef for an iterator for `element_table`.*

## Protected Attributes

- std::map< std::string, int, [string\\_less\\_than](#) > [element\\_table](#)  
*A map containing the proton numbers organized by element name.*
- std::string [element\\_list](#) [[nelements](#)]  
*The list of elements organized by proton number.*

## Static Protected Attributes

- static const int [nelements](#) = 119  
*The number of elements (proton number).*

## Data Structures

- struct [string\\_less\\_than](#)  
*String comparison operator for `element_table`.*

## 5.23.2 Member Function Documentation

### 5.23.2.1 int get\_nucleus (int Z, int N, nucleus & n) [inline]

Fill `n` with the information from [nucleus](#) with the given neutron and proton number.

All masses are given in  $\text{fm}^{-1}$ . The total mass (withouth the electrons) is put in [part::m](#) and [part::ms](#), the binding energy is placed in [nucleus::be](#), the mass excess in [nucleus::mex](#) and the degeneracy ([part::g](#)) is arbitrarily set to 1 for even A nuclei and 2 for odd A nuclei.

Definition at line 151 of file `nuclear_mass.h`.

### 5.23.2.2 virtual double binding\_energy (int Z, int N) [inline, virtual]

Return the binding energy in MeV.

The binding energy is defined to be negative for bound nuclei, thus the binding energy per baryon of Pb-208 is about  $-8 \times 208 = -1664$  MeV.

Definition at line 198 of file `nuclear_mass.h`.

### 5.23.2.3 virtual double binding\_energy\_d (double Z, double N) [inline, virtual]

Return the binding energy in MeV.

The binding energy is defined to be negative for bound nuclei, thus the binding energy per baryon of Pb-208 is about  $-8 \times 208 = -1664$  MeV.

Definition at line 212 of file `nuclear_mass.h`.

### 5.23.2.4 int eltoZ (std::string el) [inline]

Return Z given the element name.

Properly handle errors

Definition at line 242 of file nuclear\_mass.h.

#### 5.23.2.5 std::string Ztoel (size\_t Z) [inline]

Return the element name given Z.

Note that Ztoel (0) returns "n" indicating the neutron and if the argument Z is greater than 118, an empty string is returned.

Definition at line 253 of file nuclear\_mass.h.

#### 5.23.2.6 int parse\_elstring (std::string ela, int & Z, int & N, int & A) [inline]

Parse a string of the form "Pb208".

Note that this does not correctly interpret dashes, e.g. "Pb-208". will not work.

#### Idea for future

Properly ignore dashes and other non-alphanumeric characters.

Definition at line 270 of file nuclear\_mass.h.

The documentation for this class was generated from the following file:

- nuclear\_mass.h

## 5.24 nuclear\_mass::string\_less\_than Struct Reference

```
#include <nuclear_mass.h>
```

### 5.24.1 Detailed Description

String comparison operator for element\_table.

Definition at line 302 of file nuclear\_mass.h.

#### Public Member Functions

- bool **operator()** (const std::string s1, const std::string s2) const

The documentation for this struct was generated from the following file:

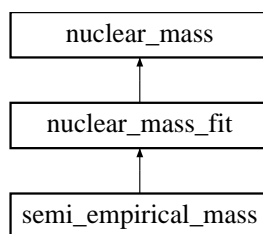
- nuclear\_mass.h

## 5.25 nuclear\_mass\_fit Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for nuclear\_mass\_fit::





### 5.25.1 Detailed Description

Fittable mass formula.

Nuclear mass formulas which are descendants of this class can be fit to experiment using [mass\\_fit](#).

Definition at line 340 of file nuclear\_mass.h.

#### Public Member Functions

- virtual int [fit\\_fun](#) (size\_t nv, const **ovector\_view** &x)  
*Fix parameters from an array for fitting.*
- virtual int [guess\\_fun](#) (size\_t nv, **ovector\_view** &x)  
*Fill array with guess from present values for fitting.*

#### Data Fields

- size\_t [nfit](#)  
*Number of fitting parameters.*

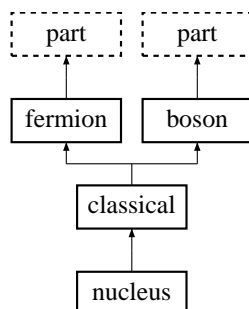
The documentation for this class was generated from the following file:

- nuclear\_mass.h

## 5.26 nucleus Class Reference

```
#include <nucleus.h>
```

Inheritance diagram for nucleus::



### 5.26.1 Detailed Description

A simple [nucleus](#) class.

The variable `part::m` is typically used for the mass of the `nucleus` with no electrons.

The binding energy of the `nucleus` (`be`) is typically defined as the mass of the `nucleus` (without the electrons) minus  $Z$  times the mass of the proton minus  $N$  times the mass of the neutron.

The mass excess (`be`) is defined as the mass of the `nucleus` including the electron contribution minus  $A$  times the mass of the atomic mass unit.

The variable `part::inc_rest_mass` is set to `false` by default, to insure that energies and chemical potentials do not include the rest mass. This is typically appropriate for nuclei.

Definition at line 51 of file `nucleus.h`.

## Data Fields

- int `Z`  
*Proton number.*
- int `N`  
*Neutron number.*
- int `A`  
*Atomic number.*
- double `mex`  
*Mass excess.*
- double `be`  
*Binding energy (with a minus sign for bound nuclei).*

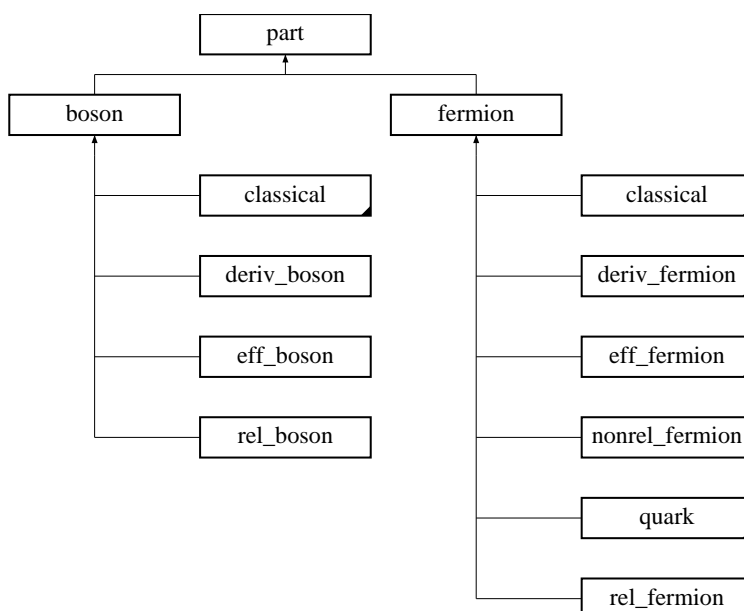
The documentation for this class was generated from the following file:

- `nucleus.h`

## 5.27 part Class Reference

```
#include <part.h>
```

Inheritance diagram for `part::`:



### 5.27.1 Detailed Description

Particle base class.

Calculate the properties of particles from their chemical potential ([calc\\_mu\(\)](#) and [pair\\_mu\(\)](#)) or from the density ([calc\\_density\(\)](#) and [pair\\_density\(\)](#)).

When non-interacting is false, the thermodynamic integrals need both a value of "mu" and "nu". "nu" is an effective chemical potential which appears in the argument of the exponential of the Fermi-function.

Keep in mind, that the pair functions use [anti\(\)](#), which assumes that  $\nu \rightarrow -\nu$  and  $\mu \rightarrow -\mu$  for the anti-particles, which might not be true for interacting particles. When non-interacting is true, then "ms" is set equal to "m", and "nu" is set equal to "mu", everywhere.

The "density" functions use the value of nu (or mu when non\_interacting is true) for an initial guess. Zero is very likely a bad guess, but these functions will not warn you about this.

Definition at line 100 of file part.h.

#### Public Member Functions

- [part](#) (double [m](#)=0.0, double [g](#)=0.0)  
*make a particle of mass m and degeneracy g.*
- virtual int [init](#) (double [m](#), double [g](#))  
*Set the mass m and degeneracy g.*
- virtual int [calc\\_mu](#) (const double temper)  
*Calculate properties as function of chemical potential.*
- virtual int [calc\\_density](#) (const double temper)  
*Calculate properties as function of density.*
- virtual int [pair\\_mu](#) (const double temper)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int [pair\\_density](#) (const double temper)  
*Calculate properties with antiparticles as function of density.*
- virtual int [anti](#) ([part](#) &ax)  
*Make an anti-particle.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("part").*

#### Data Fields

- double [g](#)  
*degeneracy*
- double [m](#)  
*mass*
- double [n](#)  
*density*
- double [ed](#)  
*energy density*
- double [pr](#)  
*pressure*
- double [mu](#)  
*chemical potential*
- double [en](#)  
*entropy*
- double [ms](#)  
*effective mass (Dirac unless otherwise specified)*
- double [nu](#)  
*effective chemical potential*

- bool `inc_rest_mass`  
*derivative of energy with respect to effective mass*
- bool `non_interacting`  
*When this is true, `nu` and `ms` are set equal to `mu` and `m` by `calc_mu()`, etc.. (default true).*
- std::string `name`  
*The name usually defaults to the class name.*

The documentation for this class was generated from the following file:

- `part.h`

## 5.28 part\_ioc Class Reference

```
#include <part_ioc.h>
```

### 5.28.1 Detailed Description

Setup I/O for particle classes.

Definition at line 44 of file `part_ioc.h`.

#### Data Fields

- `part_io_type * part_io`
- `thermo_io_type * thermo_io`
- `quark_io_type * quark_io`
- `rel_boson_io_type * rel_boson_io`
- `rel_fermion_io_type * rel_fermion_io`
- `boson_io_type * boson_io`
- `classical_io_type * classical_io`
- `eff_boson_io_type * eff_boson_io`
- `eff_fermion_io_type * eff_fermion_io`
- `eff_quark_io_type * eff_quark_io`
- `fermion_io_type * fermion_io`
- `nonrel_fermion_io_type * nonrel_fermion_io`

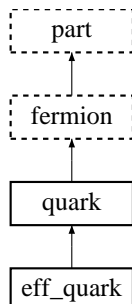
The documentation for this class was generated from the following file:

- `part_ioc.h`

## 5.29 quark Class Reference

```
#include <quark.h>
```

Inheritance diagram for `quark::`



### 5.29.1 Detailed Description

Quark base.

Definition at line 43 of file quark.h.

#### Public Member Functions

- `quark` (double mass=0.0, double dof=0.0)  
*Create a `quark` with mass `m` and degeneracy `g`.*
- virtual int `calc_mu` (const double temper)  
*Calculate properties as function of chemical potential.*
- virtual int `calc_density` (const double temper)  
*Calculate properties as function of density.*
- virtual int `pair_mu` (const double temper)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int `pair_density` (const double temper)  
*Calculate properties with antiparticles as function of density.*
- virtual const char \* `type` ()  
*Return string denoting type ("quark").*

#### Data Fields

- double `B`  
*Contribution to the bag constant.*
- double `qq`  
*Quark condensate.*

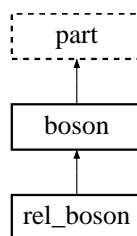
The documentation for this class was generated from the following file:

- quark.h

## 5.30 rel\_boson Class Reference

```
#include <rel_boson.h>
```

Inheritance diagram for rel\_boson::



### 5.30.1 Detailed Description

Equation of state for a relativistic `boson`.

#### Todo

Testing not completely finished.

Definition at line 48 of file rel\_boson.h.

## Public Member Functions

- `rel_boson` (double `m`=0.0, double `g`=0.0)  
*Create a `boson` with mass `m` and degeneracy `g`.*
- virtual int `calc_mu` (const double `temper`)  
*Calculate properties as function of chemical potential.*
- virtual int `calc_density` (const double `temper`)  
*Calculate properties as function of density.*
- virtual int `pair_mu` (const double `temper`)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int `nu_from_n` (const double `temper`)  
*Calculate effective chemical potential from density.*
- int `set_inte` (`inte`< void \*, `funct`< void \* > > &`l_nit`, `inte`< void \*, `funct`< void \* > > &`l_dit`)  
*Set `inte` object.*
- int `set_density_root` (`root`< void \*, `funct`< void \* > > &`rp`)  
*Set the solver for use in calculating the chemical potential from the density.*
- virtual const char \* `type` ()  
*Return string denoting type ("rel\_boson").*

## Data Fields

- int `mroot_err`  
*The error value from `mroot`.*
- int `inte_err`  
*The error value from `inte`.*
- `cern_mroot_root`< void \*, `funct`< void \* > > `def_density_root`  
*The default solver for `calc_density()`.*
- `gsl_inte_qagiu`< void \*, `funct`< void \* > > `def_nit`  
*Default nondegenerate integrator.*
- `gsl_inte_qag`< void \*, `funct`< void \* > > `def_dit`  
*Default degenerate integrator.*

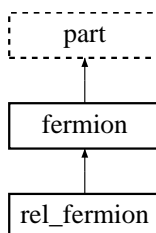
The documentation for this class was generated from the following file:

- `rel_boson.h`

## 5.31 rel\_fermion Class Reference

```
#include <rel_fermion.h>
```

Inheritance diagram for `rel_fermion`:



### 5.31.1 Detailed Description

Equation of state for a relativistic [fermion](#).

This implements an equation of state for a relativistic [fermion](#) using direct integration. Define the degeneracy parameter

$$\psi = (\nu - m^*)/T$$

where  $\nu$  is the effective chemical potential and  $m^*$  is the effective mass. For  $\psi$  greater than [deg\\_limit](#) (degenerate regime), a finite interval integrator is used and for  $\psi$  less than [deg\\_limit](#) (non-degenerate regime), an integrator over the interval from  $[0, \infty)$  is used. The upper limit on the degenerate integration is given by

$$\sqrt{(20T + \nu)^2 - m^{*,2}}$$

The default integrators are [gsl\\_inte\\_qag](#) (for degenerate particles) and [gsl\\_inte\\_qagiu](#) (for non-degenerate particles)

One way to improve the accuracy of the computation is just to decrease the tolerances on the default integration objects. This can be done, using, for example

```
rel_fermion rf(1.0, 2.0);
rf.def_dit.tolx/=1.0e2;
rf.def_dit.tolf/=1.0e2;
rf.def_nit.tolx/=1.0e2;
rf.def_nit.tolf/=1.0e2;
```

which decreases the both the relative and absolute tolerances for both the degenerate and non-degenerate integrators. If one is using either the [calc\\_density\(\)](#) or [pair\\_density\(\)](#) functions, one may also have to improve the accuracy of the solver which determines the chemical potential from the density. For the default solver, this could be done with

```
rf.def_density_root.tolx/=1.0e2;
rf.def_density_root.tolf/=1.0e2;
```

Of course if these tolerances are too small, the calculation may fail, in which case the reported uncertainties are not necessarily correct.

#### Note:

This does not work with `inc_rest_mass=false`

#### Idea for future

Allow the user to change the upper limit on the degenerate integration and the hard-coded value of 200 in the integrands.

#### Idea for future

It appears this doesn't compute the uncertainty in the chemical potential or density with [calc\\_density\(\)](#). This could be fixed.

Definition at line 98 of file `rel_fermion.h`.

### Public Member Functions

- [rel\\_fermion](#) (double `m`=0.0, double `g`=0.0)  
*Create a [fermion](#) with mass `m` and degeneracy `g`.*
- virtual int [calc\\_mu](#) (const double `temper`)  
*Calculate properties as function of chemical potential.*
- virtual int [calc\\_density](#) (const double `temper`)  
*Calculate properties as function of density.*
- virtual int [pair\\_mu](#) (const double `temper`)

*Calculate properties with antiparticles as function of chemical potential.*

- virtual int [pair\\_density](#) (const double temper)  
*Calculate properties with antiparticles as function of density.*
- virtual int [nu\\_from\\_n](#) (const double temper)  
*Calculate effective chemical potential from density.*
- int [set\\_inte](#) (inte< void \*, funct< void \* > > &non\_it, inte< void \*, funct< void \* > > &deg\_it)  
*Set integrators.*
- int [set\\_density\\_root](#) (root< void \*, funct< void \* > > &rp)  
*Set the solver for use in calculating the chemical potential from the density.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("rel\_fermion").*

## Data Fields

- double [deg\\_limit](#)  
*The critical degeneracy at which to switch integration techniques.*
- [fermion\\_unc](#)  
*Storage for the uncertainty.*
- bool [guess\\_from\\_nu](#)  
*If true, use the present value of the chemical potential as a guess for the new chemical potential.*
- [cern\\_mrroot\\_root](#)< void \*, funct< void \* > > [def\\_density\\_root](#)  
*The default solver for [calc\\_density\(\)](#).*
- [gsl\\_inte\\_qag](#)< void \*, funct< void \* > > [def\\_dit](#)  
*The default integrator for degenerate fermions.*
- [gsl\\_inte\\_qagiu](#)< void \*, funct< void \* > > [def\\_nit](#)  
*The default integrator for non-degenerate fermions.*

## Protected Member Functions

- double [density\\_fun](#) (double u, void \*&pa)  
*The integrand for the density for non-degenerate fermions.*
- double [energy\\_fun](#) (double u, void \*&pa)  
*The integrand for the energy density for non-degenerate fermions.*
- double [entropy\\_fun](#) (double u, void \*&pa)  
*The integrand for the entropy density for non-degenerate fermions.*
- double [deg\\_density\\_fun](#) (double u, void \*&pa)  
*The integrand for the density for degenerate fermions.*
- double [deg\\_energy\\_fun](#) (double u, void \*&pa)  
*The integrand for the energy density for degenerate fermions.*
- double [deg\\_entropy\\_fun](#) (double u, void \*&pa)  
*The integrand for the entropy density for degenerate fermions.*
- int [solve\\_fun](#) (double x, double &yy, void \*&pa)  
*Solve for the chemical potential given the density.*
- int [pair\\_fun](#) (double x, double &yy, void \*&pa)  
*Solve for the chemical potential given the density with antiparticles.*

## Protected Attributes

- [inte](#)< void \*, funct< void \* > > \* [nit](#)  
*The non-degenerate integrator.*
- [inte](#)< void \*, funct< void \* > > \* [dit](#)  
*The degenerate integrator.*
- [root](#)< void \*, funct< void \* > > \* [density\\_root](#)  
*The solver for [calc\\_density\(\)](#).*

The documentation for this class was generated from the following file:

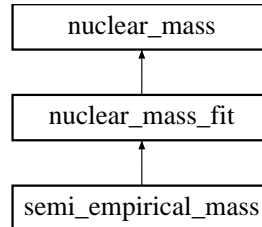


- rel\_fermion.h

## 5.32 semi\_empirical\_mass Class Reference

```
#include <nuclear_mass.h>
```

Inheritance diagram for semi\_empirical\_mass::



### 5.32.1 Detailed Description

Semi-empirical mass formula.

A simple semi-empirical mass formula of the form

$$E/A = B + S_s \frac{1}{A^{1/3}} + E_c \frac{Z^2}{A^{4/3}} + S_v \left(1 - \frac{2Z}{A}\right)^2 + E_{\text{pair}}(Z, N)$$

where

$$E_{\text{pair}}(Z, N) = \frac{E_{\text{pair}}}{A^{3/2}} \times \begin{cases} -1 & \text{N and Z even} \\ +1 & \text{N and Z odd} \\ 0 & \text{otherwise} \end{cases}$$

#### Note:

The default parameters are arbitrary, and are not determined from a fit.

Definition at line 383 of file nuclear\_mass.h.

### Public Member Functions

- virtual double [mass\\_excess\\_d](#) (double Z, double N)  
*Given Z and N, return the mass excess in MeV.*
- virtual int [fit\\_fun](#) (size\_t nv, const **ovector\_view** &x)  
*Fix parameters from an array for fitting.*
- virtual int [guess\\_fun](#) (size\_t nv, **ovector\_view** &x)  
*Fill array with guess from present values for fitting.*

### Data Fields

- double [B](#)  
*Binding energy (negative and in MeV, default -16).*
- double [Sv](#)  
*Symmetry energy (in MeV, default 23.7).*
- double [Ss](#)  
*Surface energy (in MeV, default 18).*
- double [Ec](#)

*Coulomb energy (in MeV, default 0.7).*

- double [Epair](#)

*Pairing energy (MeV, default 13.0).*

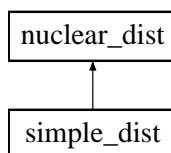
The documentation for this class was generated from the following file:

- nuclear\_mass.h

## 5.33 simple\_dist Class Reference

```
#include <nuclear_dist.h>
```

Inheritance diagram for simple\_dist::



### 5.33.1 Detailed Description

A simple nuclear distribution given a range in A and Z.

The iterator for this distribution begins with the [nucleus](#) with the lowest Z and A, and increases A before incrementing Z and beginning again with the lowest A for that value of Z. In other words, it proceeds through all the isotopes of an element first, and then proceeds to the next element.

For example, to create a **collection** of isotopes of Carbon, Nitrogen and Oxygen using the most recent (2003) Atomic Mass Evaluation, and then output the nuclei in the **collection**

```
ame_mass ame;
simple_dist fd(6,8,2,30,&ame);
for(nuclear_dist::iterator ndi=fd.begin();ndi!=fd.end();ndi++) {
    cout << ndi->Z << " " << ndi->A << " " << ndi->m << endl;
}
```

#### Todo

This takes a [nuclear\\_mass](#) pointer as input. It should probably be a reference instead? (3/18/08 - Yeah, I think it should be a reference.)

#### Todo

Add error checking in constructors and set functions

Definition at line 166 of file nuclear\_dist.h.

### Public Member Functions

- [simple\\_dist](#) (int minZ, int maxZ, int minA[], int maxA[], [nuclear\\_mass](#) \*nm)  
*Create a distribution from ranges in A specified for each Z.*
- [simple\\_dist](#) (int minZ, int maxZ, int minA, int maxA, [nuclear\\_mass](#) \*nm)  
*Create a square distribution in A and Z.*
- virtual [iterator](#) [begin](#) ()

*The beginning of the distribution.*

- virtual `iterator end()`

*The end of the distribution.*

- virtual `size_t size()`

*The number of nuclei in the distribution.*

- `int set_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass *nm)`

*Set the distribution from ranges in A specified for each Z.*

- `int set_dist(int minZ, int maxZ, int minA, int maxA, nuclear_mass *nm)`

*Set a square distribution in A and Z.*

### 5.33.2 Constructor & Destructor Documentation

#### 5.33.2.1 `simple_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass *nm)`

Create a distribution from ranges in A specified for each Z.

The length of the arrays minA and maxA should be exactly  $\text{maxZ} - \text{minZ} + 1$ .

### 5.33.3 Member Function Documentation

#### 5.33.3.1 `int set_dist(int minZ, int maxZ, int minA[], int maxA[], nuclear_mass *nm)`

Set the distribution from ranges in A specified for each Z.

The length of the arrays minA and maxA should be exactly  $\text{maxZ} - \text{minZ} + 1$ .

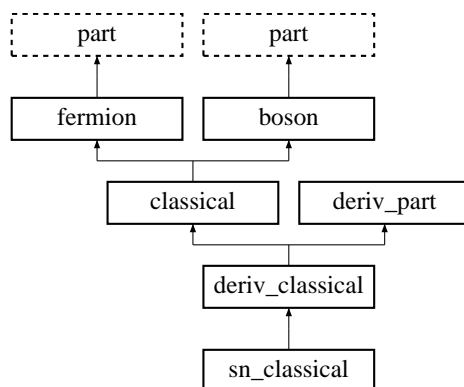
The documentation for this class was generated from the following file:

- `nuclear_dist.h`

## 5.34 sn\_classical Class Reference

```
#include <sn_classical.h>
```

Inheritance diagram for `sn_classical`:



### 5.34.1 Detailed Description

Equation of state for a `classical` particle with derivatives.

#### Todo

This does not work with `inc_rest_mass=true`

Definition at line 42 of file `sn_classical.h`.

### Public Member Functions

- `sn_classical` (double `m`=0.0, double `g`=0.0)  
*Create a [fermion](#) with mass `m` and degeneracy `g`.*
- virtual int `calc_mu` (const double `temper`)  
*Calculate properties as function of chemical potential.*
- virtual int `calc_density` (const double `temper`)  
*Calculate properties as function of density.*
- virtual int `pair_mu` (const double `temper`)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int `pair_density` (const double `temper`)  
*Calculate properties with antiparticles as function of density.*
- virtual const char \* `type` ()  
*Return string denoting type ("`sn_classical`").*

### 5.34.2 Member Function Documentation

#### 5.34.2.1 virtual int `pair_mu` (const double *temper*) [virtual]

Calculate properties with antiparticles as function of chemical potential.

#### Bug

This function probably doesn't work (3/6/08). See end of `eff_fermion_ts.cpp`.

Reimplemented from [classical](#).

#### 5.34.2.2 virtual int `pair_density` (const double *temper*) [virtual]

Calculate properties with antiparticles as function of density.

#### Bug

This function probably doesn't work (3/6/08). See end of `eff_fermion_ts.cpp`.

Reimplemented from [classical](#).

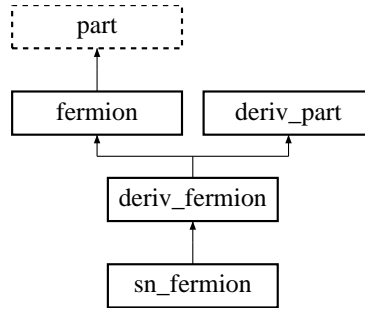
The documentation for this class was generated from the following file:

- `sn_classical.h`

## 5.35 `sn_fermion` Class Reference

```
#include <sn_fermion.h>
```

Inheritance diagram for `sn_fermion`:



### 5.35.1 Detailed Description

Equation of state for a relativistic [fermion](#).

#### Note:

This class does not work with `inc_rest_mass=true`.

This implements an equation of state for a relativistic [fermion](#) using direct integration. After subtracting the rest mass from the chemical potentials, the distribution function is

$$\left\{ 1 + \exp[(\sqrt{k^2 + m^{*2}} - m - \nu)/T] \right\}^{-1}$$

where  $k$  is the momentum,  $\nu$  is the effective chemical potential,  $m$  is the rest mass, and  $m^*$  is the effective mass. For later use, we define  $E^* = \sqrt{k^2 + m^{*2}}$ . The degeneracy parameter is

$$\psi = (\nu + (m - m^*))/T$$

For  $\psi$  greater than [deg\\_limit](#) (degenerate regime), a finite interval integrator is used and for  $\psi$  less than [deg\\_limit](#) (non-degenerate regime), an integrator over the interval from  $[0, \infty)$  is used. Typical choices are Gauss-Legendre integration for the degenerate regime and Gauss-Laguerre integration for the non-degenerate regime. The upper limit on the degenerate integration is given by the solution of

$$(\sqrt{k^2 + m^{*,2}} - m - \nu)/T = \text{flimit}$$

which is

$$\sqrt{(m + \mathcal{L})^2 - m^{*2}}$$

where  $\mathcal{L} \equiv \text{flimit} \times T + \nu$ .

In the non-degenerate regime, we make the substitution  $u = k/T$  to ensure that the variable of integration does not have units.

Uncertainties are given in [unc](#).

#### Todo

This needs to be corrected to calculate  $\sqrt{k^2 + m^{*2}} - m$  gracefully when  $m^* \approx m$ .

#### Todo

Call error handler if `inc_rest_mass` is true or update to properly treat the case when `inc_rest_mass` is true.

### Evaluation of the derivatives

The relevant derivatives of the distribution function are

$$\frac{\partial f}{\partial T} = f(1 - f) \frac{E^* - m - \nu}{T^2}$$

$$\begin{aligned}\frac{\partial f}{\partial \nu} &= f(1-f) \frac{1}{T} \\ \frac{\partial f}{\partial k} &= -f(1-f) \frac{k}{E^* T} \\ \frac{\partial f}{\partial m^*} &= -f(1-f) \frac{m^*}{E^* T}\end{aligned}$$

We also need the derivative of the entropy integrand w.r.t. the distribution function, which is

$$\mathcal{S} \equiv f \ln f + (1-f) \ln(1-f) \quad \frac{\partial \mathcal{S}}{\partial f} = \ln \left( \frac{f}{1-f} \right) = \left( \frac{\nu - E^* + m}{T} \right)$$

where the entropy density is

$$s = -\frac{g}{2\pi^2} \int_0^\infty \mathcal{S} k^2 dk$$

The derivatives can be integrated directly ([method = direct](#)) or they may be converted to integrals over the distribution function through an integration by parts ([method = byparts](#))

$$\int_a^b f(k) \frac{dg(k)}{dk} dk = f(k)g(k)|_{k=a}^{k=b} - \int_a^b g(k) \frac{df(k)}{dk} dk$$

using the distribution function for  $f(k)$  and 0 and  $\infty$  as the limits, we have

$$\frac{g}{2\pi^2} \int_0^\infty \frac{dg(k)}{dk} f dk = \frac{g}{2\pi^2} \int_0^\infty g(k) f(1-f) \frac{k}{E^* T} dk$$

as long as  $g(k)$  vanishes at  $k = 0$ . Rewriting,

$$\frac{g}{2\pi^2} \int_0^\infty h(k) f(1-f) dk = \frac{g}{2\pi^2} \int_0^\infty f \frac{T}{k} \left[ h' E^* - \frac{h E^*}{k} + \frac{h k}{E^*} \right] dk$$

as long as  $h(k)/k$  vanishes at  $k = 0$ .

### Explicit forms

1) The derivative of the density wrt the chemical potential

$$\left( \frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{T} f(1-f) dk$$

Using  $h(k) = k^2/T$  we get

$$\left( \frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty \left( \frac{k^2 + E^{*2}}{E^*} \right) f dk$$

2) The derivative of the density wrt the temperature

$$\left( \frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2(E^* - m - \nu)}{T^2} f(1-f) dk$$

Using  $h(k) = k^2(E^* - \nu)/T^2$  we get

$$\left( \frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f}{T} \left[ 2k^2 + E^{*2} - E^* (\nu + m) - k^2 \left( \frac{\nu + m}{E^*} \right) \right] dk$$

3) The derivative of the entropy wrt the chemical potential

$$\left( \frac{ds}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - m - \nu)}{T^2} dk$$

This verifies the Maxwell relation

$$\left(\frac{ds}{d\mu}\right)_T = \left(\frac{dn}{dT}\right)_\mu$$

4) The derivative of the entropy wrt the temperature

$$\left(\frac{ds}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - m - \nu)^2}{T^3} dk$$

Using  $h(k) = k^2(E^* - \nu)^2/T^3$

$$\left(\frac{ds}{dT}\right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f(E^* - m - \nu)}{E^* T^2} [E^{*3} + 3E^* k^2 - (E^{*2} + k^2)(\nu + m)] dk$$

5) The derivative of the density wrt the effective mass

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = -\frac{g}{2\pi^2} \int_0^\infty \frac{k^2 m^*}{E^* T} f(1-f) dk$$

Using  $h(k) = -(k^2 m^*)/(E^* T)$  we get

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = -\frac{g}{2\pi^2} \int_0^\infty m^* f dk$$

The dsdT integration doesn't work well if the system is very degenerate. When method is byparts, the integral involves a large cancellation between the regions from  $k \in (0, \text{ulimit}/2)$  and  $k \in (\text{ulimit}/2, \text{ulimit})$ . Switching to method=direct and setting the lower limit to llimit, may help, but recent testing on this gave negative values for dsdT. For very degenerate systems, an expansion is probably better than trying to perform the integration.

### Todo

This class will have difficulty with extremely degenerate or extremely non-degenerate systems.

### Todo

Create a more intelligent method for dealing with bad initial guesses for the chemical potential in `calc_density()`.

Definition at line 225 of file sn\_fermion.h.

### Method of computing derivatives

- int `method`  
*Method (default is `byparts`).*
- static const int `direct` = 1  
*In the form containing  $f(1-f)$ .*
- static const int `byparts` = 2  
*Integrate by parts.*

### Public Member Functions

- `sn_fermion` (double `m`=0.0, double `g`=0.0)  
*Create a `fermion` with mass `m` and degeneracy `g`.*
- virtual int `calc_mu` (const double `temper`)  
*Calculate properties as function of chemical potential.*
- virtual int `calc_density` (const double `temper`)  
*Calculate properties as function of density.*
- virtual int `pair_mu` (const double `temper`)

*Calculate properties with antiparticles as function of chemical potential.*

- virtual int [pair\\_density](#) (const double temper)  
*Calculate properties with antiparticles as function of density.*
- virtual int [nu\\_from\\_n](#) (const double temper)  
*Calculate effective chemical potential from density.*
- int [set\\_inte](#) (inte< void \*, funct< void \* > > &unit, inte< void \*, funct< void \* > > &udit)  
*Set inte objects.*
- int [set\\_density\\_root](#) (root< void \*, funct< void \* > > &rp)  
*Set the solver for use in calculating the chemical potential from the density.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("sn\_fermion").*

## Data Fields

- double [deg\\_limit](#)  
*The critical degeneracy at which to switch integration techniques (default 2.0).*
- double [flimit](#)  
*The limit for the Fermi functions (default 20.0).*
- [deriv\\_fermion\\_unc](#)  
*Storage for the most recently calculated uncertainties.*
- [gsl\\_inte\\_qagiu](#)< void \*, funct< void \* > > [def\\_nit](#)  
*The default integrator for the non-degenerate regime.*
- [gsl\\_inte\\_qag](#)< void \*, funct< void \* > > [def\\_dit](#)  
*The default integrator for the degenerate regime.*
- [cern\\_mroot](#)< void \*, funct< void \* > > [def\\_density\\_root](#)  
*The default solver for [npen\\_density\(\)](#) and [pair\\_density\(\)](#).*

## 5.35.2 Member Function Documentation

### 5.35.2.1 int set\_inte (inte< void \*, funct< void \* > > & unit, inte< void \*, funct< void \* > > & udit)

Set **inte** objects.

The first integrator is used for non-degenerate integration and should integrate from 0 to  $\infty$  (like [gsl\\_inte\\_qagiu](#)). The second integrator is for the degenerate case, and should integrate between two finite values.

## 5.35.3 Field Documentation

### 5.35.3.1 double flimit

The limit for the Fermi functions (default 20.0).

[sn\\_fermion](#) will ignore corrections smaller than about  $\exp(-\text{flimit})$ .

Definition at line 244 of file [sn\\_fermion.h](#).

The documentation for this class was generated from the following file:

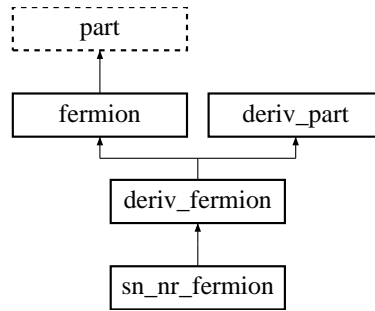
- [sn\\_fermion.h](#)

## 5.36 sn\_nr\_fermion Class Reference

```
#include <sn_nr_fermion.h>
```

Inheritance diagram for [sn\\_nr\\_fermion](#)::





### 5.36.1 Detailed Description

Equation of state for a nonrelativistic [fermion](#).

This does not include the rest mass energy in the chemical potential or the rest mass energy density in the energy density to alleviate numerical precision problems at low densities

This implements an equation of state for a nonrelativistic [fermion](#) using direct integration. After subtracting the rest mass from the chemical potentials, the distribution function is

$$\left\{ 1 + \exp \left[ \left( \frac{k^2}{2m^*} - \nu \right) / T \right] \right\}^{-1}$$

where  $\nu$  is the effective chemical potential,  $m$  is the rest mass, and  $m^*$  is the effective mass. For later use, we define  $E^* = k^2/2/m^*$ .

Uncertainties are given in [unc](#).

#### Evaluation of the derivatives

The relevant derivatives of the distribution function are

$$\begin{aligned} \frac{\partial f}{\partial T} &= f(1-f) \frac{E^* - \nu}{T^2} \\ \frac{\partial f}{\partial \nu} &= f(1-f) \frac{1}{T} \\ \frac{\partial f}{\partial k} &= -f(1-f) \frac{k}{m^* T} \\ \frac{\partial f}{\partial m^*} &= f(1-f) \frac{k^2}{2m^{*2} T} \end{aligned}$$

We also need the derivative of the entropy integrand w.r.t. the distribution function, which is quite simple

$$S \equiv f \ln f + (1-f) \ln(1-f) \quad \frac{\partial S}{\partial f} = \ln \left( \frac{f}{1-f} \right) = \left( \frac{\nu - E^*}{T} \right)$$

where the entropy density is

$$s = -\frac{g}{2\pi^2} \int_0^\infty S k^2 dk$$

The derivatives can be integrated directly or they may be converted to integrals over the distribution function through an integration by parts

$$\int_a^b f(k) \frac{dg(k)}{dk} dk = f(k)g(k) \Big|_{k=a}^{k=b} - \int_a^b g(k) \frac{df(k)}{dk} dk$$

using the distribution function for  $f(k)$  and 0 and  $\infty$  as the limits, we have

$$\frac{g}{2\pi^2} \int_0^\infty \frac{dg(k)}{dk} f dk = \frac{g}{2\pi^2} \int_0^\infty g(k) f(1-f) \frac{k}{E^* T} dk$$

as long as  $g(k)$  vanishes at  $k = 0$ . Rewriting,

$$\frac{g}{2\pi^2} \int_0^\infty h(k) f(1-f) dk = \frac{g}{2\pi^2} \int_0^\infty f \frac{Tm^*}{k} \left[ h' - \frac{h}{k} \right] dk$$

as long as  $h(k)/k$  vanishes at  $k = 0$ .

### Explicit forms

1) The derivative of the density wrt the chemical potential

$$\left( \frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{T} f(1-f) dk$$

Using  $h(k) = k^2/T$  we get

$$\left( \frac{dn}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty m^* f dk$$

2) The derivative of the density wrt the temperature

$$\left( \frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2(E^* - \nu)}{T^2} f(1-f) dk$$

Using  $h(k) = k^2(E^* - \nu)/T^2$  we get

$$\left( \frac{dn}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty \frac{f}{T} [m^*(E^* - \nu) - k^2] dk$$

3) The derivative of the entropy wrt the chemical potential

$$\left( \frac{ds}{d\mu} \right)_T = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - \nu)}{T^2} dk$$

This verifies the Maxwell relation

$$\left( \frac{ds}{d\mu} \right)_T = \left( \frac{dn}{dT} \right)_\mu$$

4) The derivative of the entropy wrt the temperature

$$\left( \frac{ds}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty k^2 f(1-f) \frac{(E^* - \nu)^2}{T^3} dk$$

Using  $h(k) = k^2(E^* - \nu)^2/T^3$

$$\left( \frac{ds}{dT} \right)_\mu = \frac{g}{2\pi^2} \int_0^\infty f \frac{m^*}{T^2} \left[ (E^* - \nu)^2 + \frac{2k^2}{m^*} (E^* - \nu) \right] dk$$

5) The derivative of the density wrt the effective mass

$$\left( \frac{dn}{dm^*} \right)_{T,\mu} = \frac{g}{2\pi^2} \int_0^\infty \frac{k^2}{2m^{*2}T} f(1-f) k^2 dk$$

Using  $h(k) = k^4/(2m^{*2}T)$  we get

$$\left( \frac{dn}{dm^*} \right)_{T,\mu} = \frac{g}{2\pi^2} \int_0^\infty f \frac{3k^2}{2m^*} dk$$

### New section

$u = k^2/2/m^*/T$  and  $y = \mu/T$ , so

$$kdk = m^* T du$$

or

$$dk = \frac{m^*T}{\sqrt{2m^*T}u} du = \sqrt{\frac{m^*T}{2u}} du$$

1) The derivative of the density wrt the chemical potential

$$\left(\frac{dn}{d\mu}\right)_T = \frac{gm^{*3/2}\sqrt{T}}{2^{3/2}\pi^2} \int_0^\infty u^{-1/2} f du$$

2) The derivative of the density wrt the temperature

$$\left(\frac{dn}{dT}\right)_\mu = \frac{gm^{*3/2}\sqrt{T}}{2^{3/2}\pi^2} \int_0^\infty f du \left[3u^{1/2} - yu^{-1/2}\right]$$

4) The derivative of the entropy wrt the temperature

$$\left(\frac{ds}{dT}\right)_\mu = \frac{gm^{*3/2}T^{1/2}}{2^{3/2}\pi^2} \int_0^\infty f \left[5u^{3/2} - 6yu^{1/2} + y^2u^{-1/2}\right] du$$

5) The derivative of the density wrt the effective mass

$$\left(\frac{dn}{dm^*}\right)_{T,\mu} = \frac{3gm^{*1/2}T^{3/2}}{2^{3/2}\pi^2} \int_0^\infty u^{1/2} f du$$

Definition at line 221 of file sn\_nr\_fermion.h.

## Public Member Functions

- [sn\\_nr\\_fermion](#) (double **m**=0.0, double **g**=0.0)  
*Create a [fermion](#) with mass **m** and degeneracy **g**.*
- virtual int [calc\\_mu](#) (const double temper)  
*Calculate properties as function of chemical potential.*
- virtual int [calc\\_density](#) (const double temper)  
*Calculate properties as function of density.*
- virtual int [pair\\_mu](#) (const double temper)  
*Calculate properties with antiparticles as function of chemical potential.*
- virtual int [pair\\_density](#) (const double temper)  
*Calculate properties with antiparticles as function of density.*
- virtual int [nu\\_from\\_n](#) (const double temper)  
*Calculate effective chemical potential from density.*
- int [set\\_density\\_root](#) (**root**< void \*, **funct**< void \* > > &rp)  
*Set the solver for use in calculating the chemical potential from the density.*
- virtual const char \* [type](#) ()  
*Return string denoting type ("sn\_nr\_fermion").*

## Data Fields

- double [flimit](#)  
*The limit for the Fermi functions (default 20.0).*
- [deriv\\_fermion\\_unc](#)  
*Storage for the most recently calculated uncertainties.*
- bool [guess\\_from\\_nu](#)  
*If true, use the present value of the chemical potential as a guess for the new chemical potential.*
- **cern\_mroot\_root**< void \*, **funct**< void \* > > [def\\_density\\_root](#)  
*The default solver for [npen\\_density\(\)](#) and [pair\\_density\(\)](#).*

## Protected Member Functions

- int [solve\\_fun](#) (double x, double &yy, void \*&pa)  
*Function to compute chemical potential from density.*
- int [pair\\_fun](#) (double x, double &yy, void \*&pa)  
*Function to compute chemical potential from density when antiparticles are included.*

## Protected Attributes

- **root**< void \*, **funct**< void \* > > \* [density\\_root](#)  
*Solver to compute chemical potential from density.*

## 5.36.2 Field Documentation

### 5.36.2.1 double flimit

The limit for the Fermi functions (default 20.0).

[sn\\_nr\\_fermion](#) will ignore corrections smaller than about  $\exp(-\text{flimit})$ .

Definition at line 235 of file `sn_nr_fermion.h`.

The documentation for this class was generated from the following file:

- `sn_nr_fermion.h`

## 5.37 thermo Class Reference

```
#include <part.h>
```

### 5.37.1 Detailed Description

A class for the thermodynamical variables (energy density, pressure, entropy density).

Definition at line 46 of file `part.h`.

## Public Member Functions

- const char \* [type](#) ()  
*Return string denoting type ("thermo").*

## Data Fields

- double [pr](#)  
*pressure*
- double [ed](#)  
*energy density*
- double [en](#)  
*entropy density*

The documentation for this class was generated from the following file:

- [part.h](#)

## 6 File Documentation

### 6.1 part.h File Reference

#### 6.1.1 Detailed Description

File for definitions for [thermo](#) and [part](#).

Definition in file [part.h](#).

```
#include <string>
#include <iostream>
#include <cmath>
#include <o2scl/constants.h>
#include <o2scl/inte.h>
#include <o2scl/collection.h>
#include <o2scl/funct.h>
#include <o2scl/mroot.h>
```

#### Data Structures

- class [thermo](#)  
*A class for the thermodynamical variables (energy density, pressure, entropy density).*
- class [part](#)  
*Particle base class.*

#### Typedefs

- typedef `io_tlate< thermo > thermo_io_type`
- typedef `io_tlate< part > part_io_type`

#### Functions

- `thermo operator+` (const [thermo](#) &left, const [thermo](#) &right)  
*Addition operator.*
- `thermo operator-` (const [thermo](#) &left, const [thermo](#) &right)  
*Subtraction operator.*
- `thermo operator+` (const [thermo](#) &left, const [part](#) &right)  
*Addition operator.*
- `thermo operator-` (const [thermo](#) &left, const [part](#) &right)  
*Subtraction operator.*

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