```
In[1]:= Off[General::"spell"]; Off[General::"spell1"];
```

The schematic Hamiltonian:

```
In[2]:= H = n (-B + K / 18 (n - n0)^2 / n0^2 + S / 162 (n - n0)^3 / n0^3 +
           G / 1944 (n - n0)^4 / n0^4 + (1 - 2 x)^2 (Sv + Svp (n - n0) / n0 + Svpp (n - n0)^2 / n0^2))
```

Out[2]= $n\left(-B+\frac{G\,(n-n0)^4}{1944\,n0^4}+\frac{K\,(n-n0)^2}{18\,n0^2}+\frac{(n-n0)^3\,S}{162\,n0^3}+\left(Sv+\frac{(n-n0)\,Svp}{n0}+\frac{(n-n0)^2\,Svpp}{n0^2}\right)(1-2\,x)^2\right)$

The compressibility:

```
In[3]:= Simplify[9 n0 D[D[H, n], n] /. n → n0 /. x → 1 / 2]
```

Out[3]= K

The new definition of the skewness:

```
In[4]:= Simplify[27 n0^3 D[D[D[H / n, n], n], n] /. n → n0 /. x → 1 / 2]
```

Out[4]= S

Define little h:

```
In[5]:= h = Simplify[((H /. x → 0) - (H /. x → 1 / 2)) / n /. n → Exp[y]]
```

Out[5]= $\frac{e^{y}\,n0\,(Svp-2\,Svpp)+e^{2\,y}\,Svpp+n0^2\,(Sv-Svp+Svpp)}{n0^2}$

Jim's Sv:

```
In[6]:= Simplify[h /. y → Log[n] /. n → n0]
```

Out[6]= Sv

Jim's Svp:

```
In[7]:= Svp1 = Simplify[D[h, y] /. y → Log[n]] /. n → n0
```

Out[7]= $\frac{n0\,(Svp-2\,Svpp)+2\,n0\,Svpp}{n0}$

Jim's Svpp:

```
In[8]:= Simplify[D[D[h, y], y] /. y → Log[n]] /. n → n0
```

Out[8]= $\frac{n0\,(Svp-2\,Svpp)+4\,n0\,Svpp}{n0}$

These don't make sense to me.

Try a new definition along Jim's line of thinking. $\delta 2$ here is just "delta squared" in a convient form to take derivatives.

```
In[9]:= Ha = H / n /. x → (1 - Sqrt[δ2]) / 2
```

Out[9]= $-B + \frac{G\,(n-n0)^4}{1944\,n0^4} + \frac{K\,(n-n0)^2}{18\,n0^2} + \frac{(n-n0)^3\,S}{162\,n0^3} + \left(Sv + \frac{(n-n0)\,Svp}{n0} + \frac{(n-n0)^2\,Svpp}{n0^2}\right)\delta 2$

```
In[10]:= D[Ha, δ2] /. n → n0
```

Out[10]= Sv

```
In[11]:= n D[D[Ha, δ2], n] /. n → n0
```

Out[11]= Svp

In[12]:= n^2 / 2 D[D[D[Ha, δ2], n], n] /. n → n0

Out[12]= Svpp

Now a final alternative, based on Brack's formula:

```
In[13]:= Hb = H /. x → (1 - δ) / 2
```

Out[13]= $n\left(-B + \frac{G\,(n-n0)^4}{1944\,n0^4} + \frac{K\,(n-n0)^2}{18\,n0^2} + \frac{(n-n0)^3\,S}{162\,n0^3} + \left(Sv + \frac{(n-n0)\,Svp}{n0} + \frac{(n-n0)^2\,Svpp}{n0^2}\right)\delta^2\right)$

```
In[14]:= D[D[Hb, δ], δ] / (2 n) /. n → n0
```

Out[14]= Sv

```
In[15]:= n D[D[D[Hb, δ], δ] / (2 n), n] /. n → n0
```

Out[15]= Svp

In[16]:= n^2 / 2 D[D[D[D[Hb, δ], δ] / (2 n), n], n] /. n → n0

Out[16]= Svpp

These two approaches are equivalent at all densities:

```
In[17]:= {D[Ha, δ2], D[D[Hb, δ], δ] / (2 n)}
```

Out[17]= $\left\{Sv + \frac{(n-n0)\,Svp}{n0} + \frac{(n-n0)^2\,Svpp}{n0^2},\ Sv + \frac{(n-n0)\,Svp}{n0} + \frac{(n-n0)^2\,Svpp}{n0^2}\right\}$

```
In[18]:= {n D[D[Ha, δ2], n], n D[D[D[Hb, δ], δ] / (2 n), n]}
```

Out[18]= $\left\{n\left(\frac{Svp}{n0} + \frac{2\,(n-n0)\,Svpp}{n0^2}\right),\ n\left(\frac{Svp}{n0} + \frac{2\,(n-n0)\,Svpp}{n0^2}\right)\right\}$

In[19]:= {n^2 / 2 D[D[D[Ha, δ2], n], n], n^2 / 2 D[D[D[D[Hb, δ], δ] / (2 n), n], n]}

Out[19]= $\left\{\frac{n^2\,Svpp}{n0^2},\ \frac{n^2\,Svpp}{n0^2}\right\}$

Now, let's try calculating S' for the schematic EOS with a different symmetry energy:

In[20]:=
```
Hsch = n (-B + K / 18 (n - n0)^2 / n0^2 + S / 162 (n - n0)^3 / n0^3 + (1 - 2 x)^2 (Sb n / n0 + Sa (n / n0)^(2/3)))
```

Out[20]=
$$n\left(-B+\frac{K\,(n-n0)^2}{18\,n0^2}+\frac{(n-n0)^3\,S}{162\,n0^3}+\left(\left(\frac{n}{n0}\right)^{2/3}Sa+\frac{n\,Sb}{n0}\right)(1-2\,x)^2\right)$$

In[21]:=
```
Simplify[n D[D[D[(Hsch /. x → (1 - δ) / 2), δ], δ] / (2 n), n] /. n → n0]
```

Out[21]=
$$\frac{2\,Sa}{3}+Sb$$

Check with a Hamiltonian with different terms in the expansion of δ:

In[22]:=
```
Hdiff = n (-B + K / 18 (n - n0)^2 / n0^2 +
    S / 162 (n - n0)^3 / n0^3 + (1 - 2 x)^2 (Sv + Svp (n - n0) / n0 + Svpp (n - n0)^2 / n0^2) +
    (1 - 2 x) (Tv + Tvp (n - n0) / n0) + (1 - 2 x)^3 (Rv + Rvp (n - n0) / n0))
```

Out[22]=
$$n\left(-B+\frac{K\,(n-n0)^2}{18\,n0^2}+\frac{(n-n0)^3\,S}{162\,n0^3}+\left(Tv+\frac{(n-n0)\,Tvp}{n0}\right)(1-2\,x)+\left(Sv+\frac{(n-n0)\,Svp}{n0}+\frac{(n-n0)^2\,Svpp}{n0^2}\right)(1-2\,x)^2+\left(Rv+\frac{(n-n0)\,Rvp}{n0}\right)(1-2\,x)^3\right)$$

In[23]:=
```
Simplify[{D[(Hdiff / n /. x → (1 - Sqrt[δ2]) / 2), δ2],
    D[D[(Hdiff /. x → (1 - δ) / 2), δ], δ] / (2 n)} /. n → n0]
```

Out[23]=
$$\left\{Sv+\frac{Tv+3\,Rv\,\delta 2}{2\sqrt{\delta 2}},\ Sv+3\,Rv\,\delta\right\}$$

In[24]:=
```
Simplify[{n^2 / 2 D[D[D[(Hdiff / n /. x → (1 - Sqrt[δ2]) / 2), δ2], n], n],
   n^2 / 2 D[D[D[D[(Hdiff /. x → (1 - δ) / 2), δ], δ] / (2 n), n], n]}]
```

Out[24]=
$$\left\{\frac{n^2\,Svpp}{n0^2},\ \frac{n^2\,Svpp}{n0^2}\right\}$$

Check with a Hamiltonian with mn != mp:

In[25]:=
```
Hiso = Mn (1 - x) n + Mp x n + n (-B + K / 18 (n - n0)^2 / n0^2 +
      S / 162 (n - n0)^3 / n0^3 + (1 - 2 x)^2 (Sv + Svp (n - n0) / n0 + Svpp (n - n0)^2 / n0^2))
```

Out[25]=
$$n\left(-B+\frac{K\,(n-n0)^2}{18\,n0^2}+\frac{(n-n0)^3\,S}{162\,n0^3}+\left(Sv+\frac{(n-n0)\,Svp}{n0}+\frac{(n-n0)^2\,Svpp}{n0^2}\right)(1-2\,x)^2\right)+Mn\,n\,(1-x)+Mp\,n\,x$$

In[26]:=
```
Simplify[{D[(Hiso / n /. x → (1 - Sqrt[δ2]) / 2), δ2],
    D[D[(Hiso /. x → (1 - δ) / 2), δ], δ] / (2 n)} /. n → n0]
```

Out[26]=
$$\left\{\frac{Mn-Mp+4\,Sv\sqrt{\delta 2}}{4\sqrt{\delta 2}},\ Sv\right\}$$

In[27]:=
```
Simplify[{n^2 / 2 D[D[D[(Hiso / n /. x → (1 - Sqrt[δ2]) / 2), δ2], n], n],
   n^2 / 2 D[D[D[D[(Hiso /. x → (1 - δ) / 2), δ], δ] / (2 n), n], n]}]
```

Out[27]=
$$\left\{\frac{n^2\,Svpp}{n0^2},\ \frac{n^2\,Svpp}{n0^2}\right\}$$

Jim’s new alternative for the Hamiltonian:

```
In[28]:= Hjnew = n (-B + K / 18 (n - n0)^2 / n0^2 + S / 162 (n - n0)^3 / n0^3 +
      G / 1944 (n - n0)^4 / n0^4 + (1 - 2 x)^2 (Sv + Svp (n - n0) / n0 + Svpp / 2 (n - n0)^2 / n0^2))
```

Out[28]= $n\left(-B+\frac{G\,(n-n0)^4}{1944\,n0^4}+\frac{K\,(n-n0)^2}{18\,n0^2}+\frac{(n-n0)^3\,S}{162\,n0^3}+\left(Sv+\frac{(n-n0)\,Svp}{n0}+\frac{(n-n0)^2\,Svpp}{2\,n0^2}\right)(1-2\,x)^2\right)$

```
In[29]:= n^2 D[D[D[(Hjnew / n /. x → (1 - Sqrt[δ2]) / 2), δ2], n], n] /. n → n0
```

```
Out[29]= Svpp
```