Manual for the supernova EOS tables – v1.04

Matthias Hempel*
Department of Physics,
University of Basel,
Klingelbergstr. 82,
4056 Basel,
Switzerland
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I. MODEL

This is the manual for the supernova equation of state (EOS) tables calculated with the statistical model with excluded volume and interactions of Hempel and Schaffner-Bielich (HS) [1] and its variants presented in Refs. [2] and [3]. In this manual we focus on the different nuclear interactions and nuclear mass tables used for the calculation of the EOS tables and the data structure and content of the tables. We do not repeat the entire description of the EOS model, but only list the changes compared to the published description [1] of the model.

List of changes

- **Maximum temperature for nuclei:** In the original version nuclei were only included up to a temperature of $T^{\rm nuc} = 20$ MeV. For larger temperatures, matter was assumed to be uniform nucleon matter, i.e. consisting of only neutrons and protons. It turned out that this "switch" was at too low temperatures, because if densities are high enough light nuclei could be still abundant for even larger temperatures (see also Ref. [4]). Thus from version v1.02 on, $T^{\rm nuc} = 50$ MeV was used.

- **Excited states:** Excited states are now only considered up to the binding energy BE of the corresponding nucleus:

$$g_{A,Z}(T) = g^0_{A,Z} + \frac{c_1}{A^{5/3}} \int_0^{E_{max}} dE^* e^{-E^*/T} \exp\left(\sqrt{2a(A)E^*}\right) \tag{1}$$

  where we use $E_{max} = BE$, to represent that the excited states still have to be bound. The inclusion of excited states up to infinite energies has only a minor influence on the composition but leads to an unphysically large contribution of excited states to the energy density and entropy.

- **Neutron-drip nuclei:** We determine the neutron drip line and eliminate all nuclei of the mass tables behind it. This concerns the experimental nuclei from Ref. [5] and from the theoretical nuclear mass tables as well (the mass tables are described in Sec. IV). We did this because nuclear structure calculations for these nuclei are not very reliable. Furthermore, this gives a consistent criterion which nuclei are considered for the calculation of the EOS.

- **Definition of electron and proton chemical potentials:** In Ref. [1] the electron chemical potential contained a contribution from the Coulomb interactions, given by: $\frac{p_{Coul}}{n_e}$. Now we include this Coulomb part of the chemical potential in the proton chemical potential, so that the stored electron chemical potential is just the normal chemical potential of the Fermi-Dirac distribution function. The same definition is used in the EOS tables of Lattimer and Swesty [6] and Shen et al. [7, 8]. Note that both definitions are physically meaningful. However, the present definition has the advantage, that the EOS is also thermodynamic consistent, if the electron contribution is not included in the table. This was not the case in the previous definition.

- **Nucleon masses:** We always use the experimental values of the nucleon masses, also within the relativistic mean-field (RMF) part of the calculation. This avoids any spurious jumps when going from the ideal gas regime

*Electronic address: matthias.hempel@unibas.ch

Quantity	Symbol	Value	Unit
fine structure constant	α	$7.2973525376\times10^{-3}$	—
speed of light	c	29979245800	cm/s
reduced Planck's constant times c	$\hbar c$	197.3269631	MeV fm
electron mass	m_e	0.51099891	MeV/c^2
neutron mass	m_n	939.565346	MeV/c^2
proton mass	m_p	938.272013	MeV/c^2

TABLE I: Values of physical constants used in the calculation.

to large densities where the RMF interactions become important. However, this represents a slight change of the interactions for some of the used RMF parameterizations. We checked that the small change of the nucleon masses to their real values is negligible for nuclear matter properties.

II. PHYSICAL CONSTANTS

For all physical constants we use the 2006 CODATA values (`www.codata.org`, `www.physics.nist.gov/cuu/Constants/archive2006.html`, [9]), which are listed in Table I. In this document, we use natural units $\hbar = c = k_B = 1$.

III. NUCLEAR INTERACTIONS

For the relativistic mean-field (RMF) interactions of the nucleons we use the parameter sets TM1 [10], TMA [11], FSUgold [12], NL3 [13], DD2 [14], SFHo and SFHx [2], and IUFSU [15]. Note that the BHBΛ and BHBΛϕ tables [3] are also based on DD2. In the following, we give a brief description of the interaction terms included in the Lagrangians and the fitting procedures the parameter sets are based on.

- **NL3:** For the parameter set NL3 binding energies and charge radii of 10 nuclei and neutron radii were included in the fit procedure. In this parameterization only cubic and quartic scalar self interactions are considered in addition to the normal couplings to the nucleons.
- **TM1 & TMA:** TM1 was developed together with TM2, which were fitted to binding energies and charge radii of light (TM2) and heavy nuclei (TM1). TMA is based on an interpolation of these two parameter sets. The coupling parameters g_i of the set TMA are chosen to be mass-number dependent of the form $g_i = a_i + b_i/A^{0.4}$, with a_i and b_i being constants, to have a good description of nuclei over the entire range of mass number. For uniform nuclear matter the couplings become constants and are given by a_i. TMA was also used in Ref. [1], where our EOS model was described in detail. The Lagrangians of TM1 and TMA have the same form, only the coupling parameters and meson masses are different. Compared to NL3, vector self-interactions are included in addition, which modify the high density behavior of the EOS.
- **FSUgold & IUFSU:** In FSUgold and IUFSU in addition the coupling between the omega- and the rho-meson is included. This leads to a different behavior of the density dependence of the symmetry energy, see e.g. [16]. The IUFSU interaction was developed further from FSUgold, by considering the neutron skin thickness of ^{208}Pb and the mass of pulsar J0348+0432 [17] in the fit of the parameters, in addition to properties of finite nuclei. It successfully reproduces various known properties of finite nuclei, nuclear matter, neutron star observations and also gives a satisfactory description of pure neutron matter.
- **DD2, BHBΛ & BHBΛϕ:** In the DD2 parameterization employed for the nucleons in these three models density-dependent couplings are used instead of meson self-interactions. This approach turns out to be more flexible and in better agreement with more microscopic approaches. The parametrization DD2 is based on the same nuclear input as the parameter set DD [18], but experimental nucleon masses are used. BHBΛϕ (BHBΛ) includes the lambda hyperon with (without) the repulsive hyperon-hyperon interaction mediated by the phi meson. For BHBΛϕ (BHBΛ) we could not obtain physical solutions of the field equations for $n_B > 1.2$ fm^{-3} ($n_B > 1$ fm^{-3}) and thus limit the tables to lower values in density.

	n_B^0 [fm^{-3}]	BE/A [MeV]	K [MeV]	K' [$\frac{\text{MeV}}{\text{fm}^3}$]	J [MeV]	L [$\frac{\text{MeV}}{\text{fm}^3}$]	m_n^*/m_n	m_p^*/m_p	M_{max} [M$_\odot$]
TM1	0.1455	-16.31	281.6	-286.5	36.95	110.99	0.6343	0.6338	2.21
TMA	0.1472	-16.03	318.2	-572.2	30.66	90.14	0.6352	0.6347	2.02
FSUgold	0.1482	-16.27	229.5	-523.9	32.56	60.43	0.6107	0.6102	1.74
NL3	0.1482	-16.24	271.5	202.6	37.39	118.49	0.5954	0.5949	2.79
DD2	0.1491	-16.02	242.7	168.7	31.67	55.03	0.5628	0.5622	2.42
SFHo	0.1583	-16.19	245.4	-467.8	31.57	47.10	0.7609	0.7606	2.06
SFHx	0.1602	-16.16	238.8	-457.2	28.67	23.18	0.7179	0.7174	2.13
IUFSU	0.1546	-16.39	231.3	-290.3	31.29	47.20	0.6096	0.6090	1.95
BHBΛ	0.1491	-16.02	242.7	168.7	31.67	55.03	0.5628	0.5622	1.95
BHB$\Lambda\phi$	0.1491	-16.02	242.7	168.7	31.67	55.03	0.5628	0.5622	2.10

TABLE II: Nuclear matter and neutron star properties of the relativistic mean field models TM1 [10], TMA [11], FSUgold [12], NL3 [13], DD2 [14], SFHo and SFHx [2], IUFSU [15], and BHBΛ and BHB$\Lambda\phi$ [3]. Listed are the saturation density n_B^0, binding energy BE/A, incompressibility K, skewness coefficient K', symmetry energy J, symmetry energy slope coefficient L, the neutron and proton effective masses m_n^* and m_p^* at saturation density divided by the corresponding rest masses, and the maximum mass M_{max} of a cold neutron star.

- **SFHo & SFHx:** These two parameter sets have been developed with particular focus on neutron star mass and radius measurements, which indicate very compact neutron stars at intermediate masses. In addition to neutron star radii, charge radii and binding energies of ^{208}Pb and ^{90}Zr have been used in the fit. The Lagrangian of the two interactions contains in total 13 non-linear couplings, which result in a very flexible density-dependence of the symmetry energy. The details of the Lagrangian are given in Ref. [19]. For SFHo we encountered problems at very high densities: for $n_B \geq 2$ fm^{-3} no physical solutions of the field equations could be obtained. Instead of changing the grid of the EOS table, we decided to set all entries to zero.

Table II lists some characteristic saturation properties of uniform nuclear matter, and the resulting maximum mass of a cold neutron star for the five different interactions. The nuclear matter properties in Table II are calculated for $T = 0$, and like it is done in the SN EOS tables, the real nucleon masses m_n and m_p are used. As noted before, this corresponds to a slight change of the parameterizations NL3, TM1, TMA, FSUgold, and IUFSU, which originally assume a common nucleon mass between 938 and 939 MeV. This explains differences to previously published nuclear matter properties, e.g. in Ref. [20]. However, this change only affect the last digits of the given values. Thus we accept this without any refitting, to obtain an EOS based on real nucleon masses for all thermodynamic conditions. The maximum masses in Table II where calculated directly with the EOS tables, for the lowest available temperature of $T = 0.1$ MeV and beta-equilibrium. In Fig. 1 we show the corresponding mass-radius relations, together with the constraints from Ref. [21] and [17].

IV. MASS TABLES

All EOS tables take into account the experimental data on nuclear masses from Audi, Wapstra, and Thibault [5]. For the masses of the experimentally unknown nuclei we take different theoretical nuclear structure calculations in form of nuclear mass tables. In the following we discuss the mass tables used in combination with the five different interactions:

- **TMA & TM1:** The TMA interactions are combined with the mass table of Geng et al. [22] which is also calculated with the relativistic mean field model TMA. Thus all nuclear interactions are consistent. The mass table lists 6969 even-even, even-odd and odd-odd nuclei, extending from ^{16}O to ^{331}100 from slightly above the proton to slightly below the neutron drip line. The nuclear binding energies are calculated under consideration of axial deformations and the pairing is included with a BCS-type δ-force. For the parametrization TM1 we do not have a suitable mass table at hand, thus we cannot avoid the minor "inconsistency" to use the table of Geng et al. [22], which is based on the TMA parameterization.

- **FSUgold & IUFSU:** For FSUgold and IUFSU we take a mass table which was calculated by X. Roca-Maza, see e.g. [23]. This table contains 1512 even-even nuclei, from the proton to the neutron drip, with $14 \leq A \leq 348$ and $8 \leq Z \leq 100$. No deformations are included and the pairing is introduced through a BCS approach with

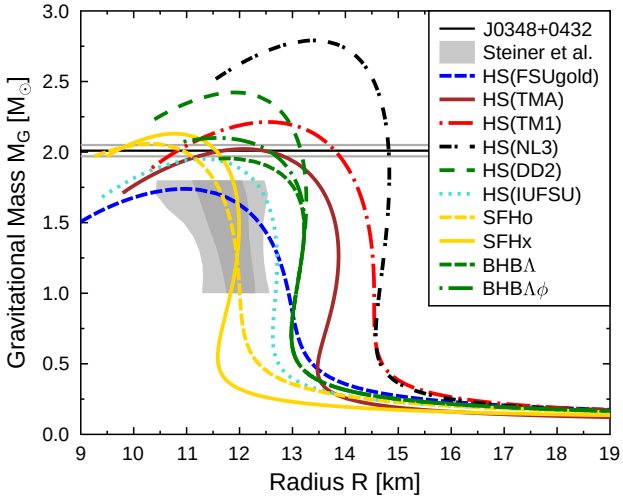

FIG. 1: The mass-radius relation for the supernova EOS tables presented in this manual at $T = 0.1$ MeV and beta-equilibrium. The horizontal lines show the constraint from pulsar PSR J0348+0432 [17]. The grey shaded regions give the one- and two-sigma constraint regions derived from neutron star observations in Ref. [21].

constant matrix elements (GA). The constant matrix element for neutrons has been fitted to reproduce the experimental binding in the tin isotopic chain and the constant matrix element for protons to the experimental binding in the $N = 82$ isotonic chain.

- **NL3:** The mass table for NL3 from Lalazissis and Raman [24] lists 1315 even-even nuclei with $10 \leq Z \leq 98$ using the BCS pairing scheme with constant pairing gap and taking into account axial deformations.
- **DD2, SFHo, SFHx, BHBΛ, & BHBΛϕ:** We do not have any nuclear structure calculations for the DD2, SFHo, and SFHx interactions. Thus we chose the mass table of the finite range droplet model (FRDM) of Ref. [25, 26] for nuclei without experimentally measured binding energies. Even today this mass model still gives one of the best reproductions of experimental binding energies, and thus very convenient for our purposes.

V. MESH OF THE EOS TABLES

Note that two different formats of the EOS tables are provided, first in an "extended format" and second in the Shen 98 format, i.e. in a format similar to the EOS tables of Ref. [7]. The different formats are explained in detail below. The EOS tables cover the following range:

temperature T

The temperature can be calculated from the temperature index $i = 1, ..., 81$ by:

$$T = 0.1 \cdot 10^{0.04(i-1)} \text{ MeV} . \tag{2}$$

The mesh considered is:
0.1 MeV $\leq T \lesssim$ 158.5 MeV; exponential mesh of $\log_{10}(\Delta T) = 0.04$, giving 81 T-values.

electron fraction Y_e

The nominal electron fraction can be calculated from the electron fraction index $j = 0, ..., 60$, respectively $j = 1, ..., 60$, by:

$$Y_e = j \cdot 0.01 . \tag{3}$$

It is equal to the total proton fraction Y_p^{tot}, due to charge neutrality. The mesh considered is:
Extended format:

- BHBΛ, & BHBΛϕ: $0.01 \leq Y_e \leq 0.6$; linear mesh of $\Delta Y_e = 0.01$, giving 60 Y_e-values.
- All others: $0 \leq Y_e \leq 0.6$; linear mesh of $\Delta Y_e = 0.01$, giving 61 Y_e-values.

Shen 98 format: $0.01 \leq Y_e \leq 0.6$; linear mesh of $\Delta Y_e = 0.01$, giving 60 Y_e-values ($Y_e = 0$ is never included).

baryon number density n_B

The nominal baryon number density can be calculated from the baryon number density index $k = 1, ..., k_{\max}$ by:

$$n_B = 10^{-12} \cdot 10^{0.04(k-1)} \text{ fm}^{-3} . \tag{4}$$

The mesh considered is:

- BHBΛ: $k_{\max} = 301$, 10^{-12} fm$^{-3} \leq n_B \leq 1$ fm^{-3}; exponential mesh of $\log_{10}(\Delta n_B) = 0.04$, giving 301 n_B-values.
- BHBΛϕ: $k_{\max} = 303$, 10^{-12} fm$^{-3} \leq n_B \lesssim 1.2$ fm^{-3}; exponential mesh of $\log_{10}(\Delta n_B) = 0.04$, giving 303 n_B-values.
- SFHo: $k_{\max} = 326$, 10^{-12} fm$^{-3} \leq n_B \leq 10$ fm^{-3}; exponential mesh of $\log_{10}(\Delta n_B) = 0.04$, giving 326 n_B-values. Note however, that all entries of SFHo are set to zero for $309 \leq k \leq 326$, because no physical solutions of the field equations could be obtained. For this EOS, the maximum density of $k = 308$ with real entries is $n_B = 1.905$ fm^{-3}, respectively $\rho_B = 3.162 \cdot 10^{15}$ g/cm^3.
- All others: $k_{\max} = 326$, 10^{-12} fm$^{-3} \leq n_B \leq 10$ fm^{-3}; exponential mesh of $\log_{10}(\Delta n_B) = 0.04$, giving 326 n_B-values.

VI. DESCRIPTION OF THE TABLES - EXTENDED FORMAT

A. Entries of the tables

The tables in the extended format contain the contribution of photons, electrons and positrons. In the following, all thermodynamic variables which are not further specified correspond to the definitions as given in Ref. [1]. For each grid point the following 19 different thermodynamic quantities are listed and denoted by the EOS entry index $l = 1, ..., 19$. Entry 19, which is the lambda mass fraction, is only available for BHBΛ and BHBΛϕ, i.e., for all other tables there are only the first 18 entries.

1. baryon number density: n_B [fm^{-3}]
   n_B is given by $n_B = n_n + n_p + \sum_{A,Z} A n_{A,Z} (+n_\Lambda)$, whereas here and in the following always $A \geq 2$ in the sum, and $n_{A,Z}$ is the number density of nucleus (A, Z). The stored value can be slightly different than the nominal value from Eq. (4), because it is determined numerically by root findings, see Sec. X.

2. total proton fraction: Y_p^{tot}
   The total proton fraction is defined by:
   $$Y_p^{tot} = \frac{n_p + \sum_{A,Z} Z n_{A,Z}}{n_B} \ . \tag{5}$$
   It is given by the sum of all protons, i.e., bound in nuclei and unbound, and is equal to the electron fraction Y_e due to electric charge neutrality. The stored value can be slightly different than the nominal value from Eq. (3), because it is determined numerically by root findings, see Sec. X.

3. total energy per baryon: E^{tot} [MeV]
   E^{tot} is the total energy per baryon including rest masses, and can be defined via the total energy density ϵ^{tot}:
   $$E^{tot} \ = \ \frac{\epsilon^{tot}}{n_B} \ . \tag{6}$$

4. total pressure: p^{tot} [MeV/fm^3]

5. total entropy per baryon: S^{tot} [k$_B$]
   The total entropy per baryon is related to the entropy density via
   $$S^{tot} \ = \ \frac{s^{tot}}{n_B} \ . \tag{7}$$

6. neutron chemical potential relative to the neutron mass: μ_n^{nonrel} [MeV]
   The neutron chemical potential is given in its non-relativistic equivalent form, i.e. without the neutron mass m_n:
   $$\mu_n^{nonrel} \ = \ \mu_n - m_n \ . \tag{8}$$
   For the BHBΛ and BHBΛϕ tables one has $\mu_\Lambda = \mu_n$.

7. proton chemical potential relative to the proton mass: μ_p^{nonrel} [MeV]
   The proton chemical potential is given in its non-relativistic equivalent form, i.e. without the proton mass m_p:
   $$\mu_p^{nonrel} \ = \ \mu_p - m_p \ , \tag{9}$$
   Note that the used definition for μ_p differs from the original one in Ref. [1]. The Coulomb part of the chemical potential is now included here in the proton chemical potential instead of in the electron chemical potential, see also Sec. I.

8. electron chemical potential relative to the electron mass: μ_e^{nonrel} [MeV]
   The electron chemical potential is given in its non-relativistic equivalent form, i.e. without the electron mass m_e:
   $$\mu_e^{nonrel} \ = \ \mu_e - m_e \ . \tag{10}$$

Compared to the original definition of the electron chemical potential in Ref. [1], the Coulomb part of the chemical potential is now included in the proton chemical potential, see also Sec. I. Thus μ_e^{nonrel} is now just the kinetic part of the chemical potential, without any contributions of interactions.

9. total free energy per baryon: F^{tot} [MeV]
The total free energy per baryon including rest masses is given by the total free energy density:

$$F^{tot} = \frac{f^{tot}}{n_B} . \tag{11}$$

10. mass fraction of unbound neutrons: X_n
The mass fraction of unbound neutrons is given by:

$$X_n = n_n/n_B . \tag{12}$$

11. mass fraction of unbound protons: X_p
The mass fraction of unbound protons is given by:

$$X_p = n_p/n_B . \tag{13}$$

12. mass fraction of deuterons: X_d
The mass fraction of deuterons ($A = 2, Z = 1$) is given by:

$$X_d = 2n_{2,1}/n_B . \tag{14}$$

13. mass fraction of tritons: X_t
The mass fraction of tritons ($A = 3, Z = 1$) is given by:

$$X_t = 3n_{3,1}/n_B . \tag{15}$$

14. mass fraction of helions: X_h
The mass fraction of helions ($A = 3, Z = 2$) is given by:

$$X_h = 3n_{3,2}/n_B . \tag{16}$$

15. mass fraction of alphas: X_α
The mass fraction of alphas ($A = 4, Z = 2$) is given by:

$$X_\alpha = 4n_{4,2}/n_B . \tag{17}$$

16. mass fraction of heavy nuclei: X_A
The mass fraction of heavy nuclei is defined by:

$$X_A = \sum_{A,Z}{}' A n_{A,Z}/n_B , \tag{18}$$

whereas the prime on the sum denotes all nuclei except neutrons, protons, deuterons, tritons, helions, alphas.

17. averaged mass number of heavy nuclei: $< A >$
The mass number of the averaged heavy nucleus is defined by:

$$< A >= \frac{\sum_{A,Z}{}' A n_{A,Z}}{\sum_{A,Z}{}' n_{A,Z}} . \tag{19}$$

$< A >$ is set to zero, if $X_A = 0$.

18. averaged charge number of heavy nuclei: $< Z >$
The charge number of the averaged heavy nucleus is defined in the analog way:

$$< Z >= \frac{\sum_{A,Z}{}' Z n_{A,Z}}{\sum_{A,Z}{}' n_{A,Z}} . \tag{20}$$

$< Z >$ is set to zero, if $X_A = 0$.

19. (only for BHBΛ & BHBΛϕ) mass fraction of lambdas: X_Λ
The mass fraction of lambdas is given by:

$$X_\Lambda = n_\Lambda/n_B . \tag{21}$$

B. Storage of the data

The EOS tables in the extended format are stored as binary files, which were created with the Intel Fortran compiler `ifort` on a 64 bit machine. We plan to provide binary tables in the `HDF5` standard in the future. The data is written as a single line. A Fortran module for the EOS is provided which contains a simple Fortran routine which reads in the table. Note that in this routine also the temperature is calculated from Eq. (2) and then stored as a separate array. Also the nominal baryon number density from Eq. (4) and the nominal electron fraction from Eq. (3) are stored as arrays. We also provide a small example program which illustrates the usage of the module and the EOS table.

VII. DESCRIPTION OF THE TABLES - SHEN 98 FORMAT

A. Entries of the tables

The information is stored in a format which is very similar to the table of Shen et al. [7, 8] so that it can easily be implemented in running codes. In these tables only the baryonic contribution is given, i.e. photons, electrons, positrons and neutrinos have to be added separately. In the following, all thermodynamic variables which are not further specified are defined analogous to Subsec. VI A, but without the electron/positron and photon contribution. For each density grid point the following 19 (respectively 20 in the cases of BHBΛ and BHBΛϕ) different thermodynamic quantities are listed:

1. logarithm of baryon mass density: $\log_{10}(\rho_B)$ [g/cm^3]
   The baryon mass density is defined as the baryon number density times the value of the atomic mass unit $m_u = 931.49432$ MeV used in Ref. [7, 8]:
   $$\rho_B = n_B m_u \ . \tag{22}$$
2. baryon number density: n_B [fm^{-3}]
3. logarithm of total proton fraction: $\log_{10}(Y_p^{tot})$
4. total proton fraction: Y_p^{tot}
5. baryonic part of the free energy per baryon relative to 938 MeV: ΔF_B [MeV]
   Like in the table of Shen et al. 938 MeV are subtracted from the baryonic part of the relativistic free energy per baryon:
   $$\Delta F_B \ = \ \frac{f_B}{n_B} - 938 \text{ MeV} \ . \tag{23}$$
6. baryonic part of the energy per baryon relative to m_u: ΔE_B [MeV]
   ΔE_B is the baryonic part of the relativistic energy per baryon relative to the atomic mass unit m_u:
   $$\Delta E_B \ = \ \frac{\epsilon_B}{n_B} - m_u \ . \tag{24}$$
7. baryonic part of the entropy per baryon S_B: [k$_B$]
   The baryonic part of the entropy per baryon is related to the baryonic entropy density via
   $$S_B \ = \ \frac{s_B}{n_B} \ . \tag{25}$$
8. average mass number of heavy nuclei: $< A >_{Z6}$
   The average mass number of heavy nuclei is defined by
   $$< A >_{Z6} = \frac{\sum_{A,Z\geq 6} A n_{A,Z}}{\sum_{A,Z\geq 6} n_{A,Z}} \ , \tag{26}$$
   where we introduced the distinction between light and heavy nuclei for the Shen 98 format by the proton number 6, i.e. carbon. $< A >_{Z6}$ is set to zero if $X_{A_{Z6}} = 0$.
9. average charge number of heavy nuclei: $< Z >_{Z6}$
   The average charge number of heavy nuclei is defined in the analog way
   $$< Z >_{Z6} = \frac{\sum_{A,Z\geq 6} Z n_{A,Z}}{\sum_{A,Z\geq 6} n_{A,Z}} \ . \tag{27}$$
   $< Z >_{Z6}$ is set to zero if $X_{A_{Z6}} = 0$.

10. effective mass: m^* [MeV]
    In the RMF calculation, the real nucleon masses m_n and m_p are used, see Sec. I. As they are not equal, also the nucleon effective masses $m_n^* = m_n + g_\sigma\sigma$ and $m_p^* = m_p + g_\sigma\sigma$ are not equal. Instead of storing these two values, we store only one "average" effective mass m^* to keep the format as close as possible to the one of Shen et al. For tables of version v1.0 we take:

$$m^* = m_{RMF} + g_\sigma\sigma \ , \tag{28}$$

    with the nucleon mass of the RMF parameterization m_{RMF}, with $m_{RMF} = 939$ MeV for FSUgold and NL3, $m_{RMF} = 938$ MeV for TM1, and $m_{RMF} = 938.9$ MeV for TMA. We want to emphasize again that m_{RMF} is not used in the calculation of the EOS, but it is only taken here as a reference value. Later we realized that it is confusing to introduce m_{RMF} as a reference value in the tables. Therefore for all versions after v1.0, we store instead the following average effective mass:

$$m^* = \frac{n_n}{n_n + n_p} m_n^* + \frac{n_p}{n_n + n_p} m_p^* \ . \tag{29}$$

11. mass fraction of unbound neutrons: X_n

12. mass fraction of unbound protons: X_p

13. mass fraction of light nuclei: $X_{a_{Z6}}$
    The mass fraction of light nuclei is defined by

$$X_{aZ6} = \sum_{A,Z \leq 5} A n_{A,Z}/n_B \ . \tag{30}$$

14. mass fraction of heavy nuclei: $X_{A_{Z6}}$
    The mass fraction of heavy nuclei is defined by

$$X_{AZ6} = \sum_{A,Z \geq 6} A n_{A,Z}/n_B \ . \tag{31}$$

15. baryonic part of the pressure: p_B [MeV/fm^3]

16. neutron chemical potential relative to the neutron mass: μ_n^{nonrel} [MeV]

17. proton chemical potential relative to the proton mass: μ_p^{nonrel} [MeV]
    Note the comments and further definitions from Subsec. VI A.

18. average mass number of light nuclei: $< a >_{Z6}$
    The average mass number of light nuclei is defined by

$$< a >_{Z6} = \frac{\sum_{A,Z \leq 5} A n_{A,Z}}{\sum_{A,Z \leq 5} n_{A,Z}} \ . \tag{32}$$

    $< a >_{Z6}$ is set to zero if $X_{a_{Z6}} = 0$.

19. average charge number of light nuclei: $< z >_{Z6}$
    The average charge number of light nuclei is defined in the analog way

$$< z >_{Z6} = \frac{\sum_{A,Z \leq 5} Z n_{A,Z}}{\sum_{A,Z \leq 5} n_{A,Z}} \ . \tag{33}$$

    $< z >_{Z6}$ is set to zero if $X_{a_{Z6}} = 0$.

20. (only for BHBΛ & BHB$\Lambda\phi$) mass fraction of lambdas: X_Λ

B. Storage of the data

The EOS tables in the Shen 98 format are stored as ASCII files. We store the three-dimensional tables in the following way: first we fix T which is noted at the top of the block, second we fix Y_p^{tot} and third n_B. The blocks with different T are divided by the line 'ccc...ccc'. A Fortran module for the EOS is provided which contains a simple Fortran routine which reads in the table in the EOS array `eos(1:t_entries,1:y_entries,1:nb_entries,1:entries)` with `t_entries` = 81, `y_entries` = 60, `entries` = 19, and `nb_entries` = 301 for BHBΛ, 303 for BHB$\Lambda\phi$ and 326 for all others. Note that in this routine by default the temperature is read in from the table (and not calculated from Eq. (2) like in the extended format) and then stored as a separate array. We also provide a small example program which illustrates the usage of the module and the EOS table.

VIII. NUCLEAR COMPOSITION

In addition to the information about the nuclear composition in the EOS tables as specified in Subsecs. VI A and VII A we provide a Fortran module which allows to calculate the number densities and mass fractions of all available nuclei. This module reads in a separate binary file which contains all the required data for the calculation. The binary file was created with the Intel Fortran compiler `ifort` on a 64 bit machine. We plan to provide binary tables in the `HDF5` standard in the future. Because most of the input data is not meaningful by itself, we do not list the entries of the data table here. The composition module contains a lot of comments and in addition an example program is provided which should explain the usage of the module and the implemented routines.

The main subroutine of the module is `sub_dist`, which calculates the nuclear distributions, the nuclear and nucleon densities and their mass fractions for given temperature index i, electron fraction index j, and baryon number density index k (see Sec. V). The mesh and its grid-points of the module correspond to the extended format of the EOS tables. The output of the module is:

- number density of unbound neutrons: n_n [fm^{-3}]
- number density of unbound protons: n_p [fm^{-3}]
- (only for BHBΛ & BHB$\Lambda\phi$) number density of lambdas: n_Λ [fm^{-3}]
- array with the number densities of all nuclei: $\{n_{A,Z}\}$ [fm^{-3}]
- mass fraction of unbound neutrons: X_n
- mass fraction of unbound protons: X_p
- (only for BHBΛ & BHB$\Lambda\phi$) mass fraction of lambdas: X_Λ
- array with the mass fractions of all nuclei: $\{X_{A,Z}\}$
  The mass fraction of nucleus (A, Z) is defined as

$$X_{A,Z} = \frac{A n_{A,Z}}{n_B} \ . \tag{34}$$

IX. DESCRIPTION OF THE TABLES WITH NUCLEON SINGLE-PARTICLE PROPERTIES

For the EOS tables of SFHo, SFHx, HS(TMA), HS(TM1), HS(FSUgold), HS(IUFSU), HS(NL3), and HS(DD2) we provide complementary ASCII tables with single-particle properties of unbound nucleons such as self-energies and effective masses. The listed quantities allow to calculate self-consistent weak charged-current interaction rates, as described in Ref. [27]. These complementary tables contain the following information:

1. baryon number density n_B [fm^{-3}]
2. total proton fraction $Y_p^{\rm tot}$ []
3. total vector self-energy of unbound neutrons $\Sigma_V^{\rm n,tot}$ [MeV]
4. total vector self-energy of unbound protons $\Sigma_V^{\rm p,tot}$ [MeV]
5. filling factor of nucleons ξ []
6. effective Dirac mass of unbound neutrons m_n^* [MeV]
7. effective Dirac mass of unbound protons m_p^* [MeV]

In combination with the information provided in the EOS tables (e.g., X_n and μ_n), it is possible to derive all quantities presented in the article of Ref. [27] and to calculate the charged-current rates. The data of the complementary tables is arranged as described in Sec. VII B. Note that for the BHBΛ and BHB$\Lambda\phi$ models, the tables are not yet available.

X. ACCURACY AND CONSISTENCY OF THE EOS TABLES

As the temperature is an input for the calculation of the tables, the values of T are exactly given by Eq. (2). Contrary, n_B and Y_p^{tot} are determined by root-findings. The calculated and stored values for n_B and the total proton density $n_B Y_p^{tot}$ (entries 1. and 2. of the EOS tables in the extended format) are allowed to have a maximum relative deviation from the precise grid-points given by Eqs. (3) and (4) of 10^{-8}. In some applications, e.g. for interpolation, it is better to have a strictly regularized grid. Therefore in the EOS modules the nominal baryon number density and the nominal electron fraction are calculated from Eq. (4), respectively Eq. (3), and then stored as separate arrays.

The modulus of the relative thermodynamic inconsistency

$$\Delta = \frac{T s^{tot} - p^{tot} + Y_p^{tot} n_B (\mu_e + \mu_p) + (1 - Y_p^{tot}) n_B \mu_n}{\epsilon^{tot}} - 1 \tag{35}$$

is everywhere below 10^{-10} for the binary tables in the extended format. For the ASCII tables in the Shen 98 format the inconsistency can be much larger because of round-off errors.

It is also checked that the mass fractions of the different particle species sum up to unity:

$$\Delta X = 1 - X_n + X_p + X_d + X_t + X_h + X_\alpha + X_A \ . \tag{36}$$

In the binary table $|\Delta X|$ is always below 10^{-11}. For the Shen 98 format,

$$\Delta X = 1 - X_n + X_p + X_{a_{Z6}} + X_{A_{Z6}} \ , \tag{37}$$

the inconsistency can again be larger, due to round-off errors.

The composition module also allows to calculate directly the baryon number density (entry 1. of the EOS tables in the extended format). The relative deviation between the baryon number density calculated with the composition module and the stored value in the EOS table is everywhere below 10^{-10}.

The accuracy and consistency of the BHBΛ & BHB$\Lambda\phi$ tables is slightly lower, and has been reported already in Ref. [3].

XI. ADDITIONAL COMMENTS

- Please note again that we assume that nuclear matter is uniform above temperatures higher or equal to 20 MeV (v1.0), i.e. consists only of neutrons and protons. From version v1.02 on, the maximum temperature was set to 50 MeV.
- For the chemical potentials we choose the true nucleon masses as the reference values, i.e., we are storing the non-relativistic equivalent form of the chemical potentials. If one uses the full relativistic chemical potentials defined as e.g. $\mu_n = \mu_n^{nonrel} + m_n$ the difference $\mu_n - \mu_p$ obviously includes the neutron to proton rest mass difference $Q = m_n - m_p$.
- Please remember that the EOS model contains a Maxwell transition from NSE to uniform nucleon matter. As we do not think that it is crucial information, we do not describe the details of the calculation of the EOS entries in the Maxwell transition region. However, we want to remark that the electron contribution to the EOS in the transition region is not given by a single ideal Fermi-Dirac gas. Thus the subtraction of the electrons from the EOS is not straightforward. In the Shen 98 format this has been done already, but I do not recommend to try to do the same for the extended format without the knowledge about the details of the calculation. Please contact me if you need different formats of the tables.
- As already remarked in Sec. III, for the SFHo interactions no solutions of the field equations could be obtained for $n_B \geq 2$ fm^{-3}. Instead of changing the grid of the corresponding EOS table, we decided to set all entries with $n_B \geq 2$ fm^{-3}, respectively $k \geq 309$, to zero.
- We wish you a lot of fun and success with the tables and highly appreciate any comments or feedback.

XII. HOW TO CITE THE TABLES

In the original HS EOS model [1], only the TMA interactions were used. The SFHo and SFHx tables have been introduced in Ref. [2], the BHBΛ & BHBΛϕ tables in Ref. [3]. The tables for TM1 and FSUgold have been applied in core-collapse supernova simulations in Refs. [20] and [2], where their properties are at least briefly discussed. Therefore these two articles can be used as references for the TM1 and FSUgold tables. DD2 was introduced first in [28], where it has been applied in simulations. This article gives also a summary of the properties and discussion of SFHo, SFHx, DD2, TM1, TMA, NL3, IUFSU, and FSUgold. For all tables, especially for the unpublished ones, additionally something like the following sentence can be used as a reference/specification:

We apply the supernova EOS table calculated with the model of Ref. [1], whereas the XX nucleon interactions [XX] and the nuclear mass table from Ref. [YY] have been used [M. Hempel, private communication].

For 'XX' the correct name and reference of the RMF parameterization of the nucleon interactions should be chosen (Sec. III), and for 'YY' the correct reference to the mass model (Sec. IV). Please write me if you have any questions.

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